



Ministero
dell'Università
e della Ricerca

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Stability of Road Vehicles

Webinar course focusing on how drivers -either human or not- lose control of the car

STAVE – Project of Relevant National Interest – P.I. Prof. G Mastinu – Prof. M Guiggiani a joint project of Politecnico di Milano and University of Pisa



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Safe vehicle handling, in any situation? – a continuous topic for vehicle dynamics engineers over time

G. Mastinu and M. Plöchl

Politecnico di Milano and Technische Universität Wien

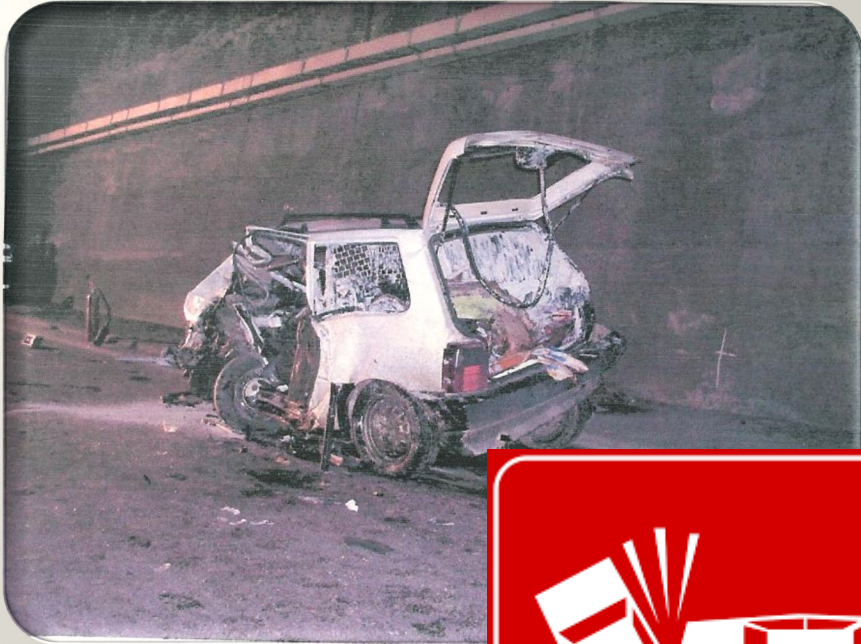
AVEC'22

Kanagawa Institute of Technology, Japan

16 September 2022

Motivation

How safe vehicle handing is related to stability ?



Motivation

Are handling/stability problems taught to engineers ?



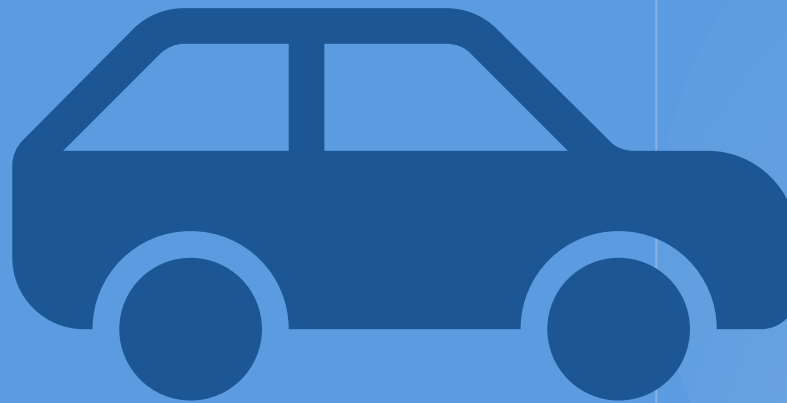
Motivation

How a stable vehicle follows the desired route/path ?

<https://www.youtube.com/watch?v=sR2PP9gUaUg>



GAPS IN CURRENT EDUCATION ON STABILITY AND HANDLING



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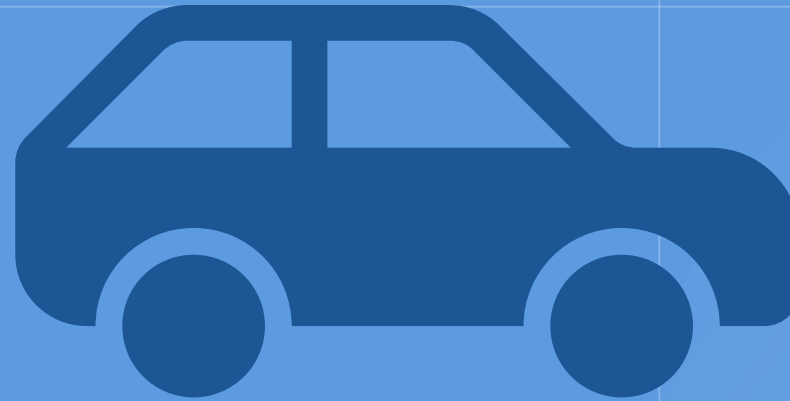
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GAP #1



SMALL DISTURBANCE



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GAP #1



**SMALL DISTURBANCE : WHAT
HAPPENS IF THE DISTURBANCE IS
BIG?**



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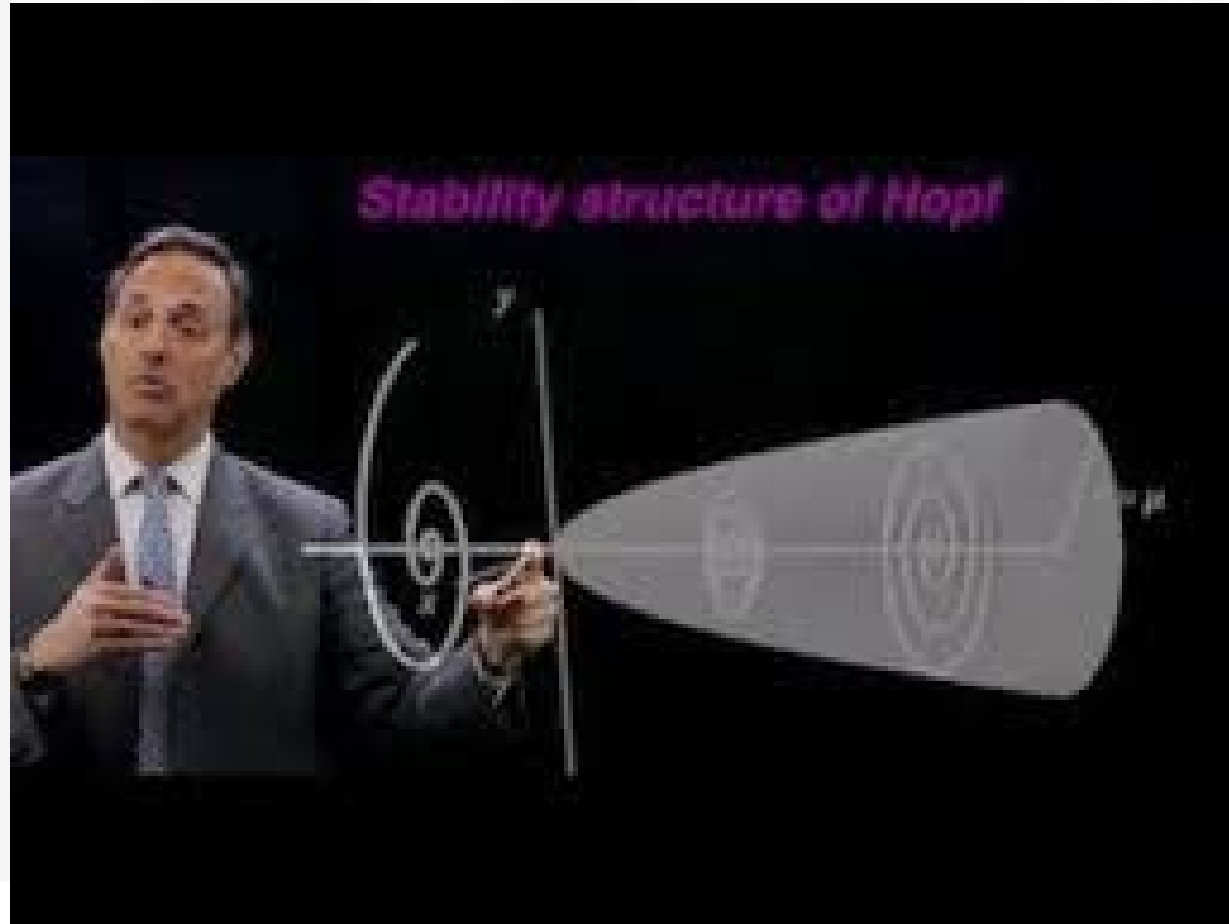


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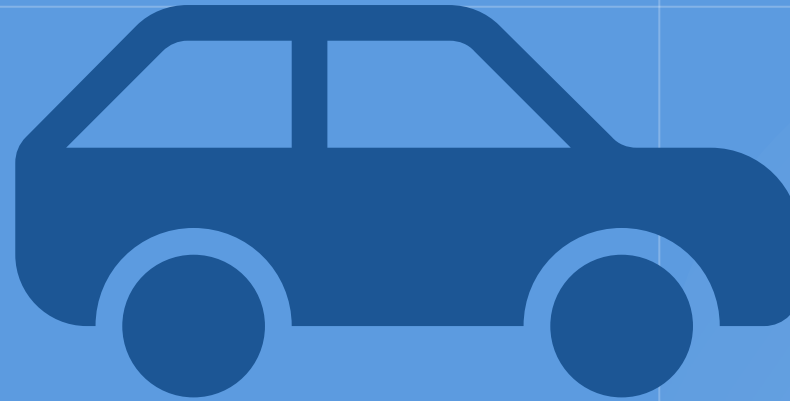
Motivation

Stability analysis and bifurcation theory provide an insight on big disturbances acting during steady state motion



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GAP #2



STABILITY OF VEHICLE ONLY



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VS



STABILITY OF VEHICLE AND DRIVER

STABILITY OF VEHICLE ONLY



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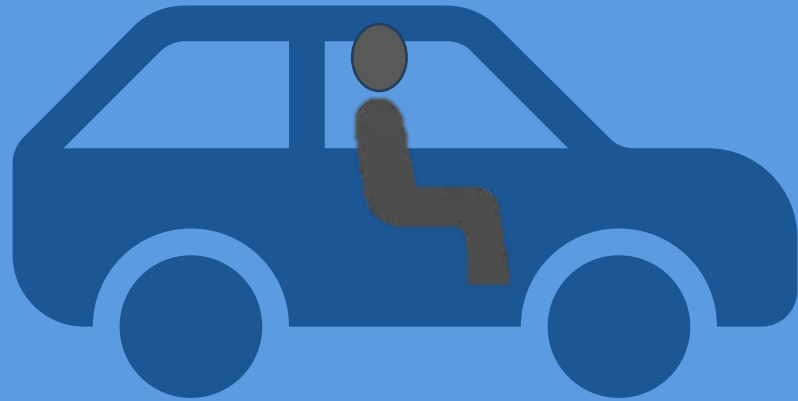


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VS



STABILITY OF VEHICLE AND DRIVER

STABILITY OF VEHICLE ONLY

**HOW THE DRIVER LOSES CONTROL
OF THE CAR?**



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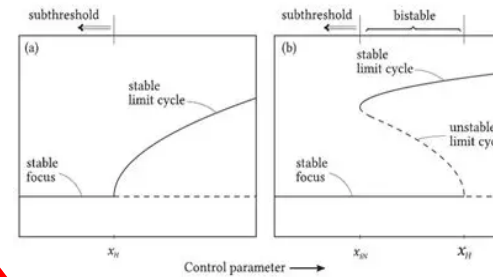
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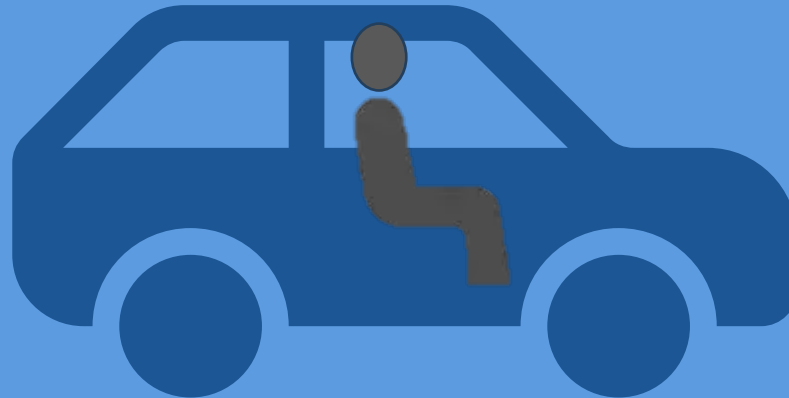
safety

comfort

LIMIT CYCLES



ROAD VEHICLE SYSTEM DYNAMICS
CORE CONCEPT: LIMIT CYCLE



efficiency



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Contents

1. Introduction
Transients and oscillatory motions related to limit cycles
2. Mechanism of instability with driver control
3. Validation: Dynamic Driving Simulator tests
4. Using limit cycles
5. Conclusions



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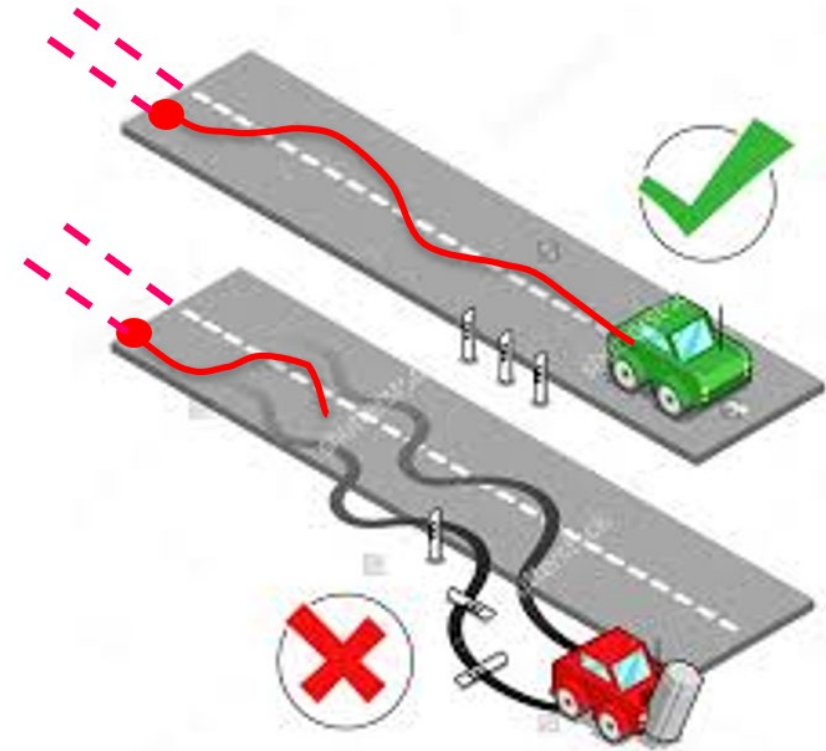
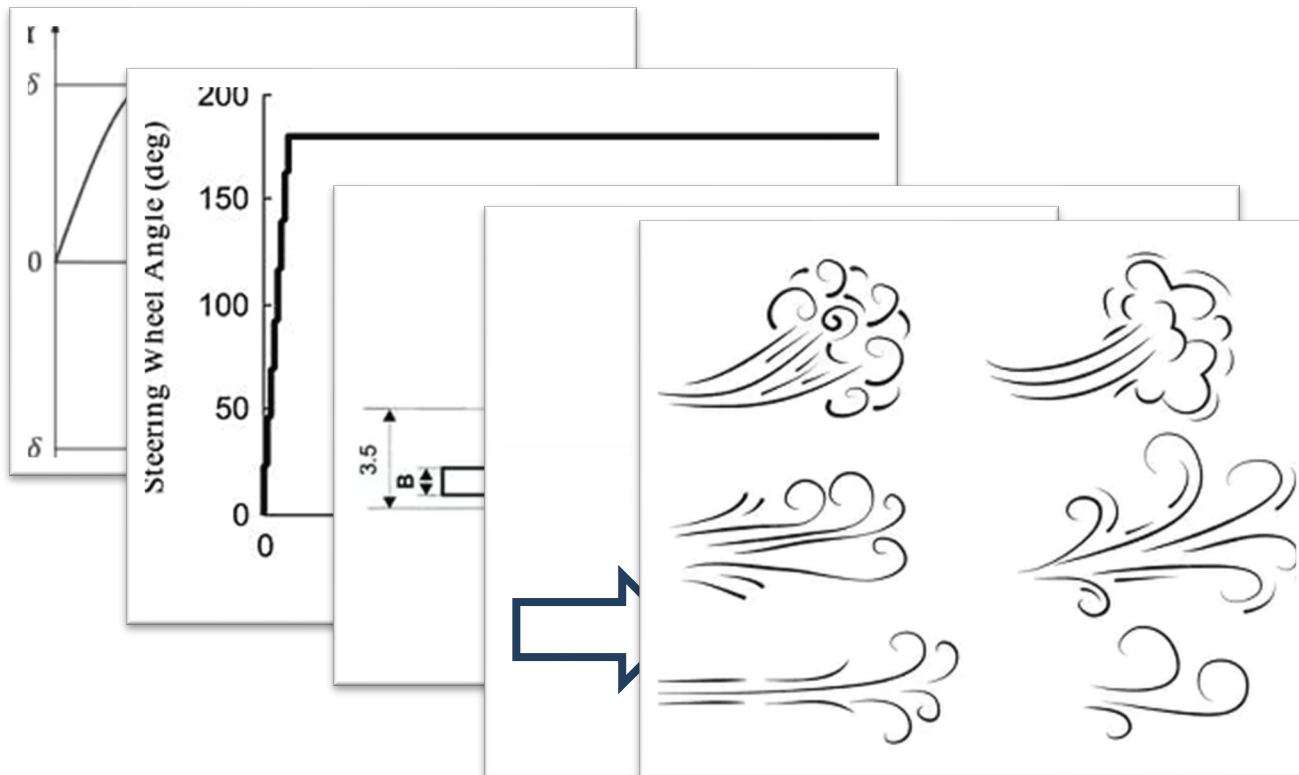
Introduction

Transients and oscillatory motions related to limit cycles

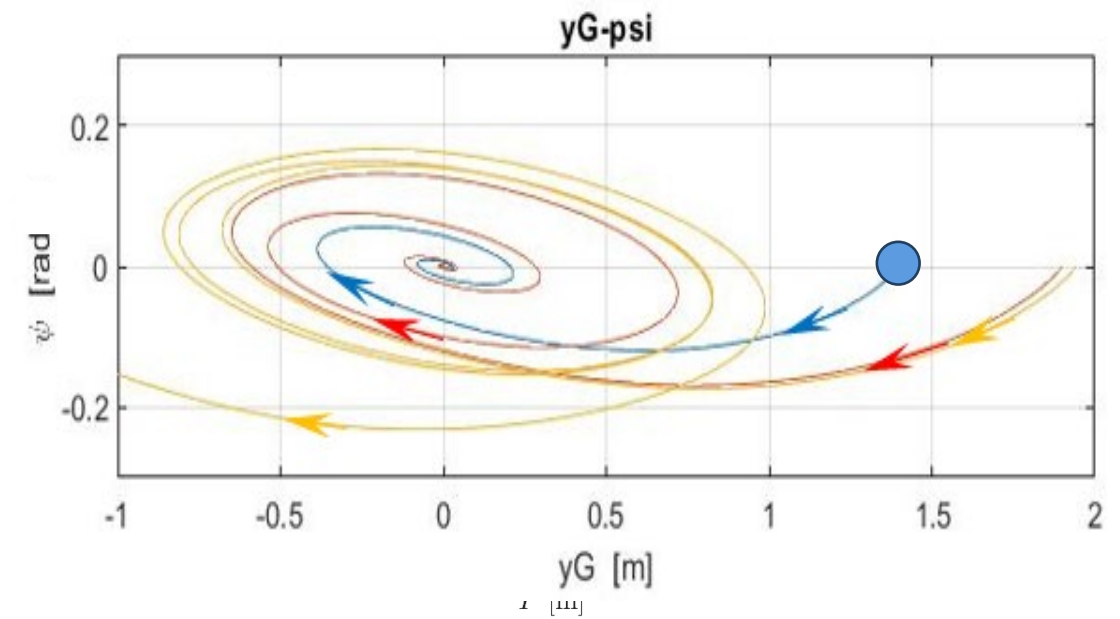
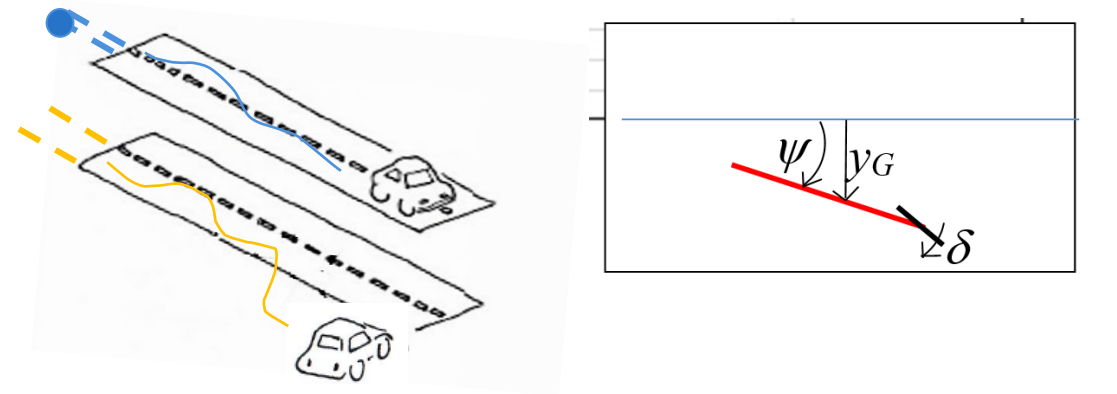
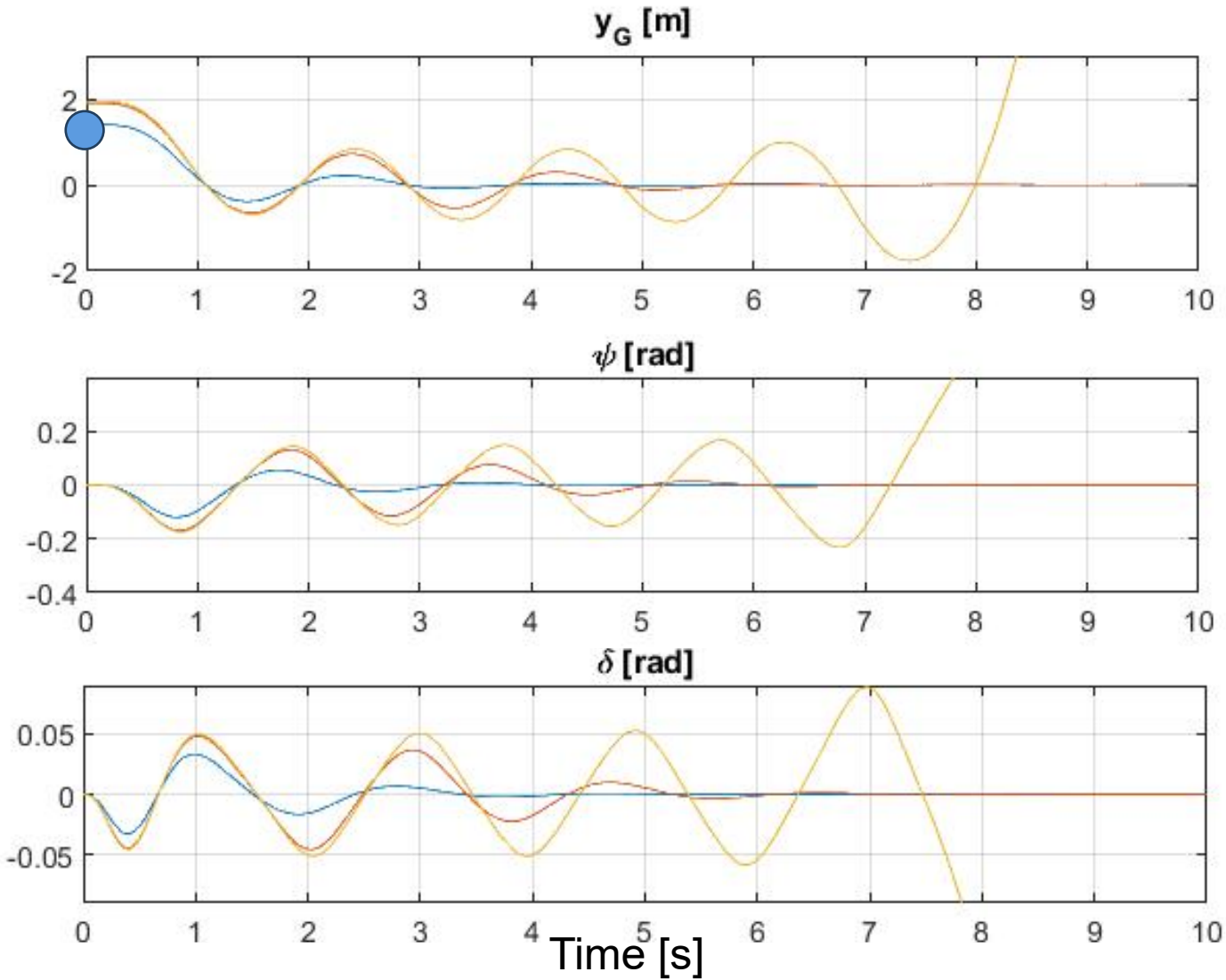
01

Transients and oscillatory motions related to are «limit cycles»

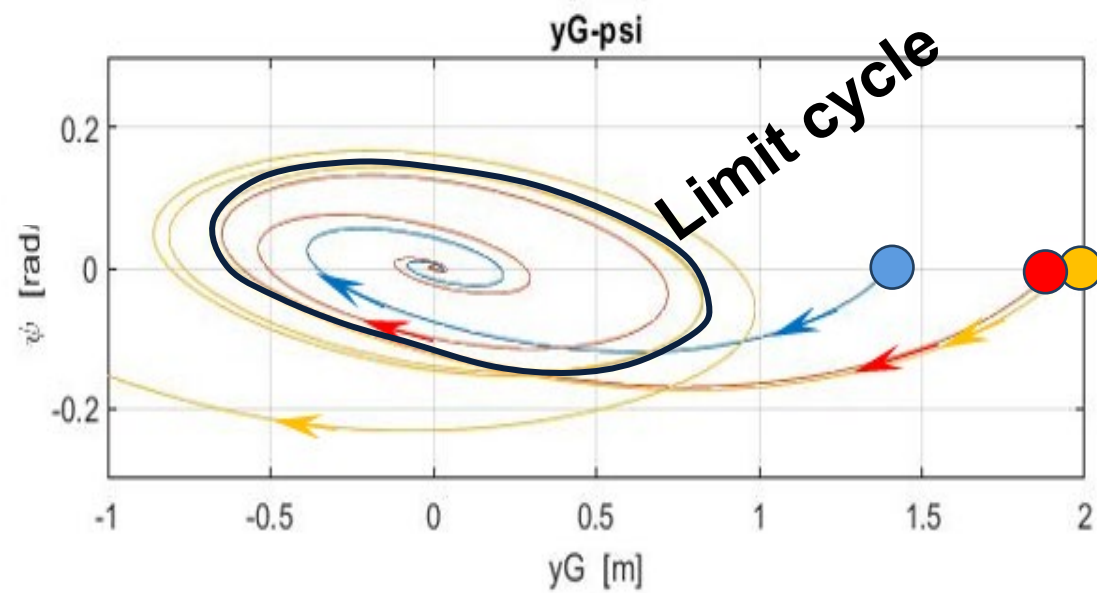
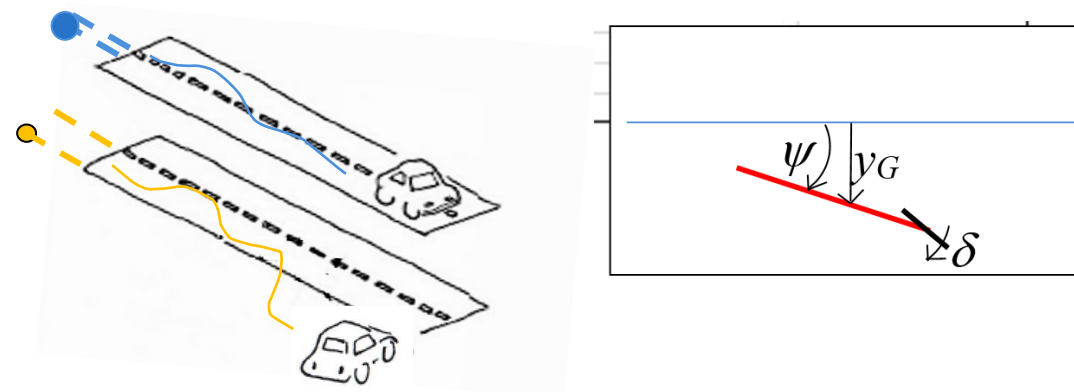
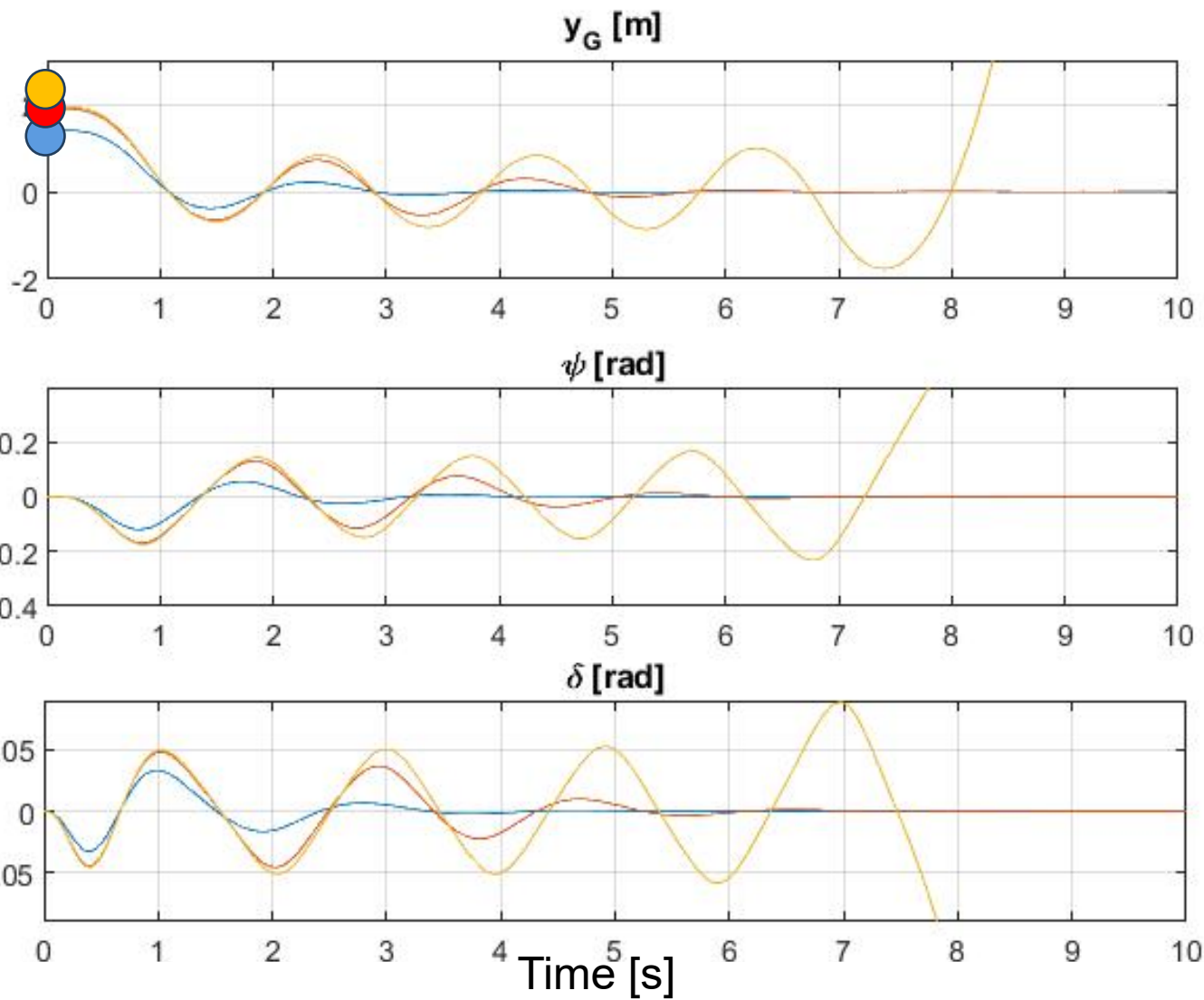
Response to ANY type of disturbance



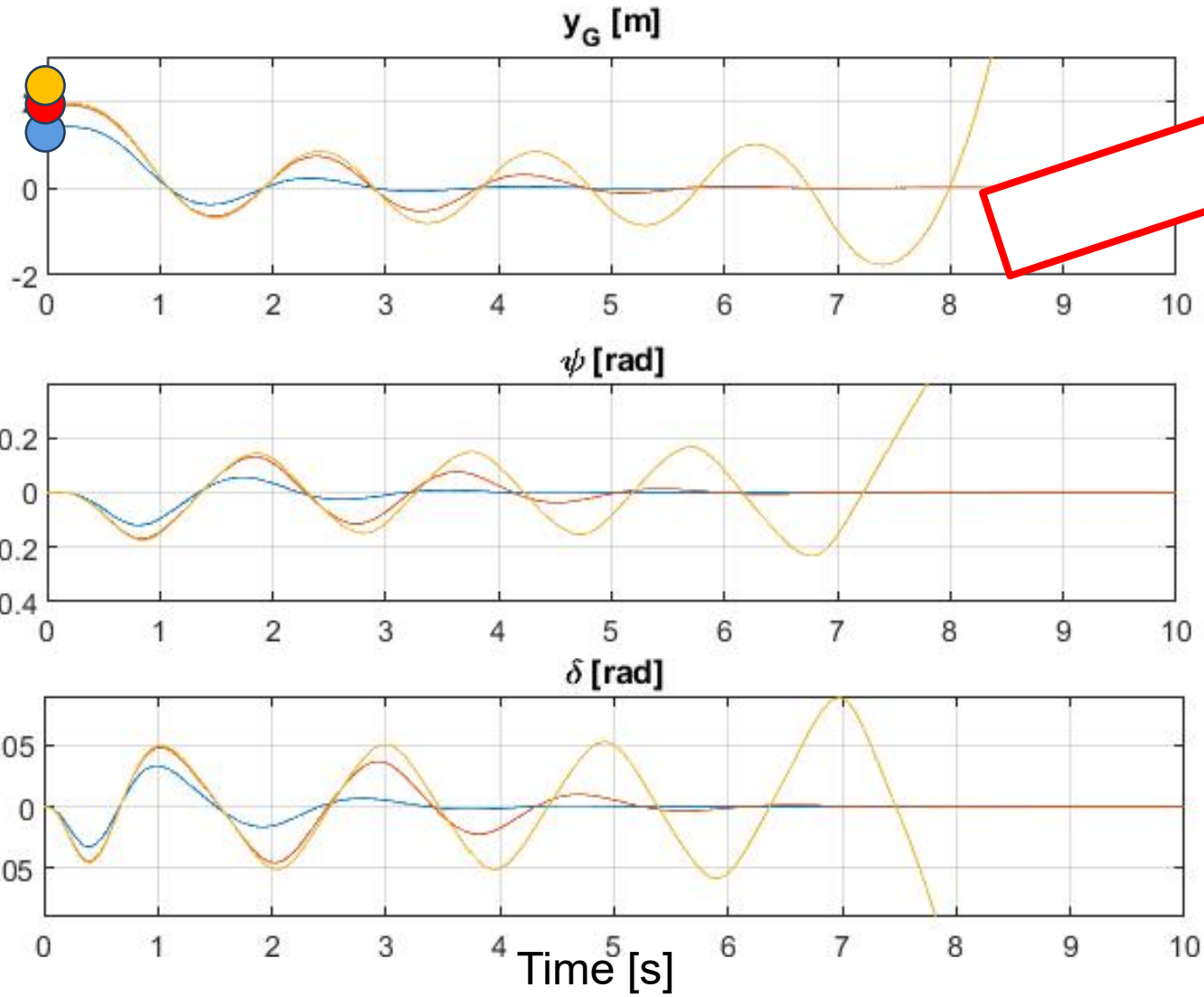
PHENOMENON - LIMIT CYCLE



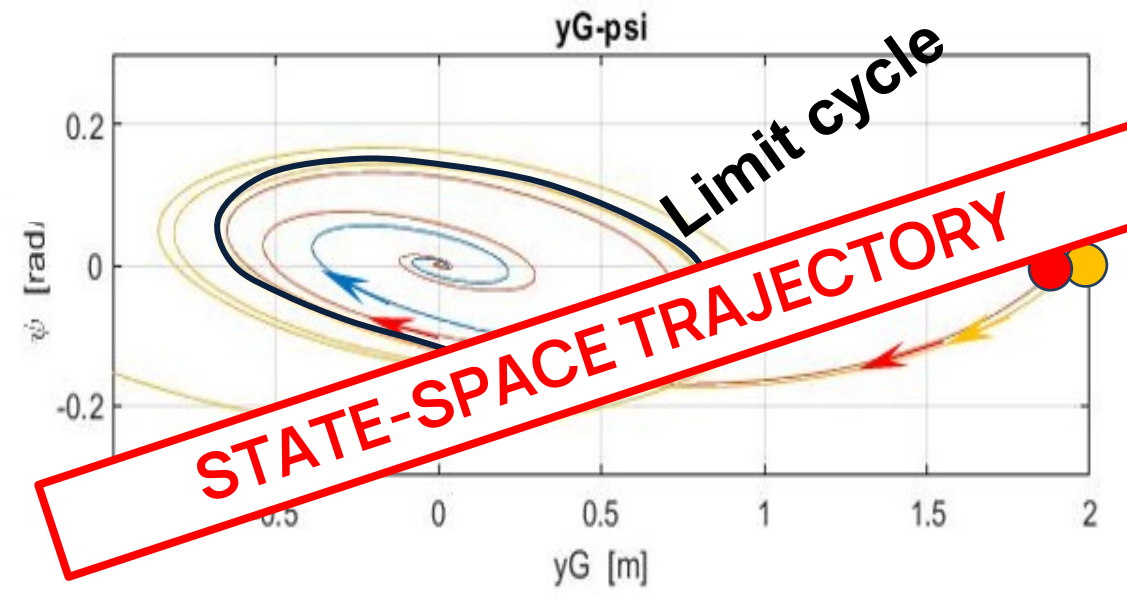
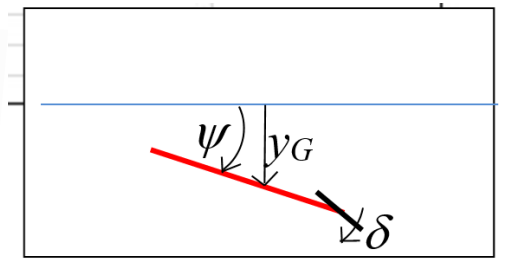
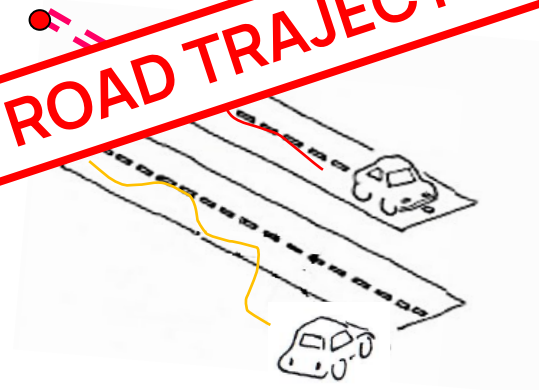
PHENOMENON - LIMIT CYCLE



PHENOMENON - LIMIT CYCLE



ROAD TRAJECTORY



STATE-SPACE TRAJECTORY

Oscillatory motions – Real scenarios



<https://video.corriere.it/video-embed/5339589e-d20a-4781-9665-9198a3aa0x1k>

Oscillatory motions – Real scenarios



<https://www.youtube.com/watch?v=-JZ-p9htCAg>

Oscillatory motions – Real scenarios



Concept #1:

LIMIT CYCLES DO EXIST AND ARE FUNDAMENTAL TO UNDERSTAND ROAD VEHICLE TRANSIENT MOTION AFTER A DISTURBANCE



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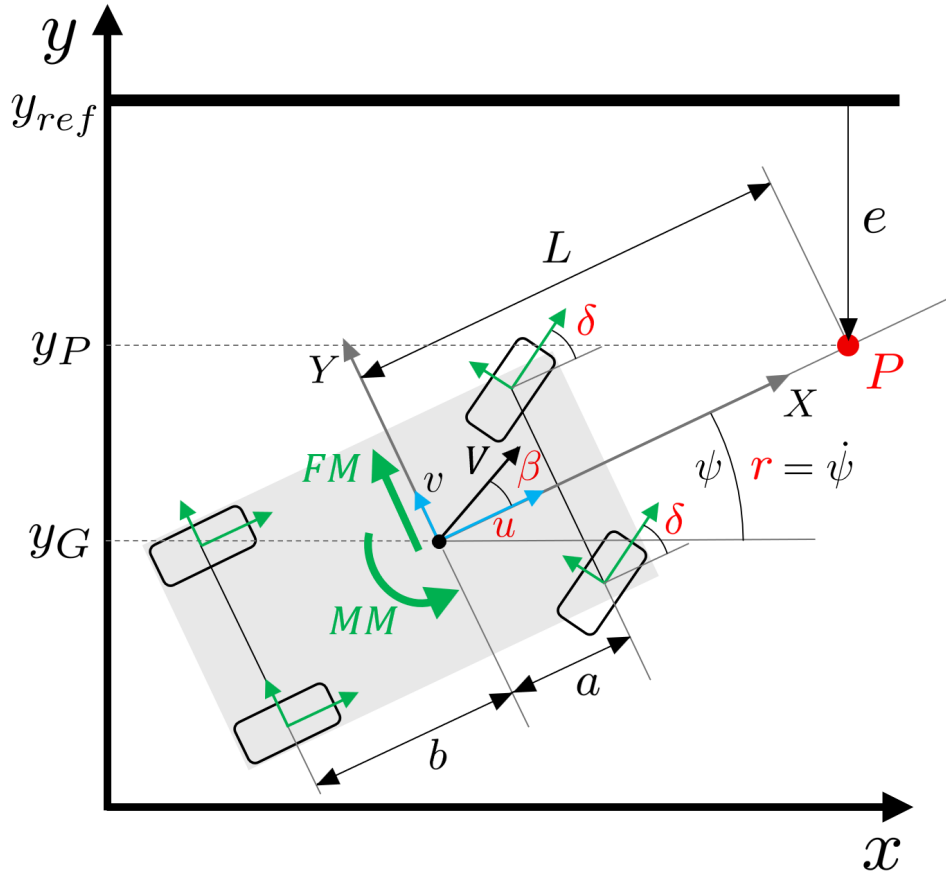
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Mechanism of instability with driver control

02

Simple vehicle and driver model



road vehicle + driver

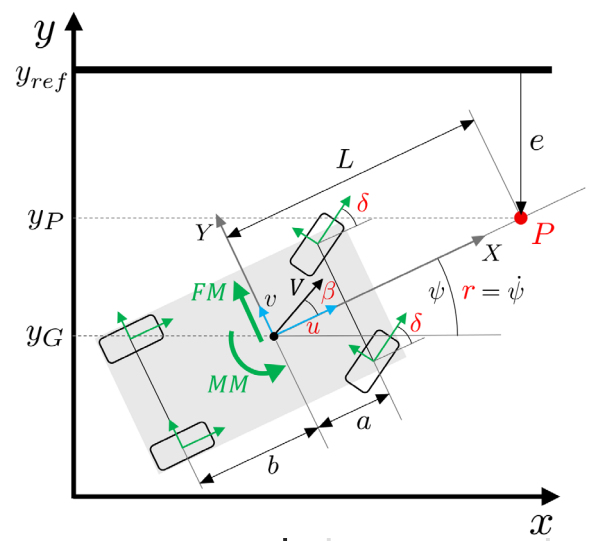
$$\left\{ \begin{array}{l} \dot{v} = \frac{1}{m} (F_{y_f} + F_{y_r}) - ur \\ \dot{r} = \frac{1}{J} (F_{y_f} a - F_{y_r} b) \\ \dot{\psi} = r \\ \dot{y}_G = u \sin(\psi) + v \cos(\psi) \\ \dot{\delta} = \delta_1 \\ \dot{\delta}_1 = \delta_2 \\ \dot{\delta}_2 = \frac{6}{\tau^3} (-\delta - \tau \delta_1 - \frac{\tau^2}{2} \delta_2 + k_p e + k_d \dot{e}) \end{array} \right. \begin{array}{l} \text{Vehicle model} \\ \\ \\ \text{Driver model} \end{array}$$

With

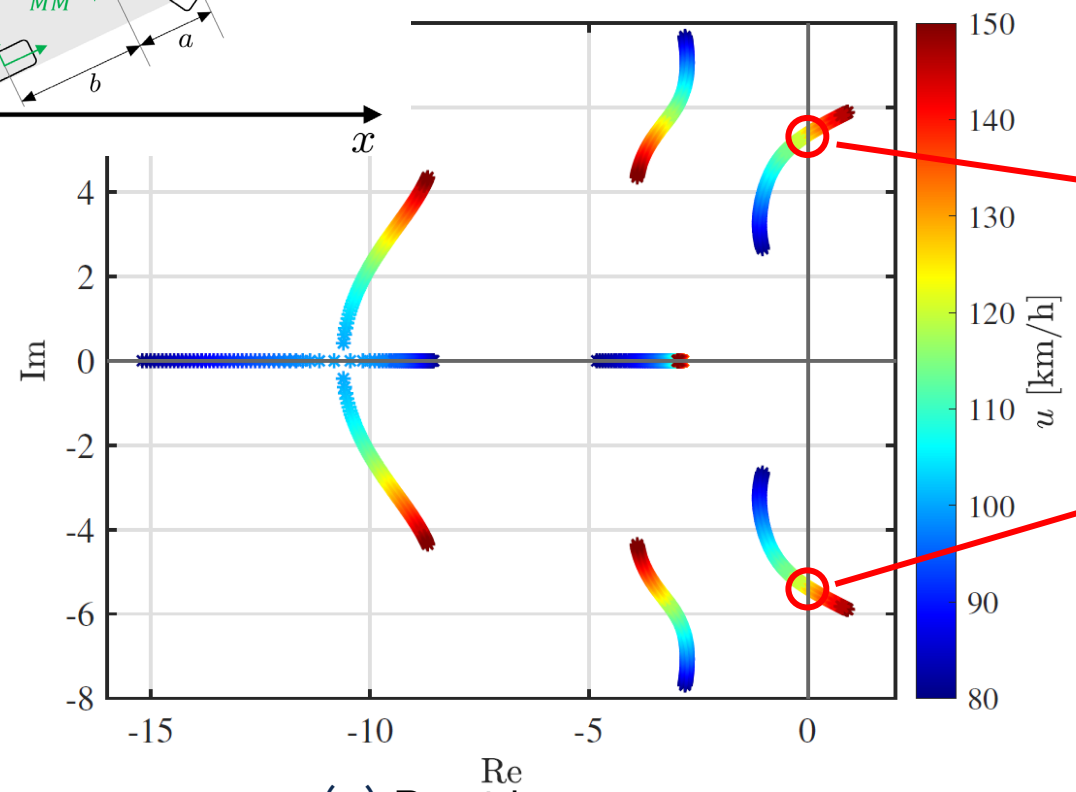
$$F_{y_i} = f(\alpha_i, B_i, C_i, D_i, E_i) \quad i = \{f, r\}$$

DDE (Delay Differential Equations) vs ODE (Ordinary Differential Equations)

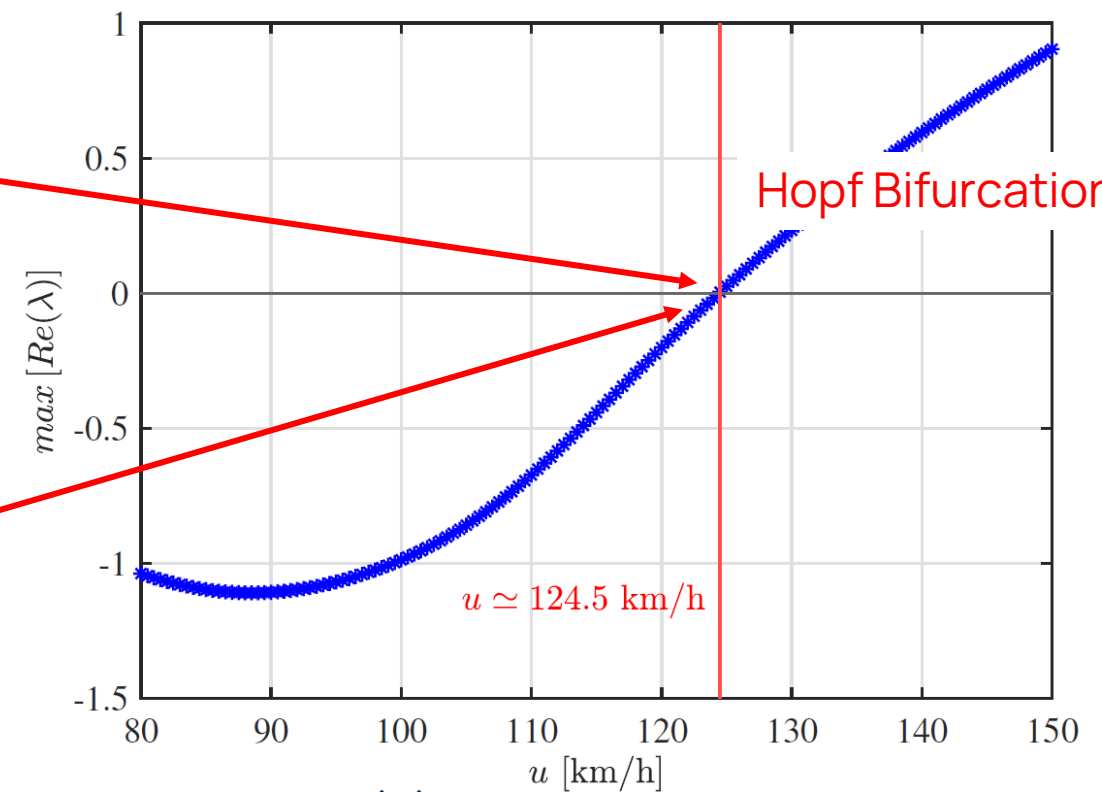
Simple vehicle and driver model – Linear Analysis



$$A(z=0) = \begin{bmatrix} -\frac{C_{\alpha,f}+C_{\alpha,r}}{mu} & -\frac{aC_{\alpha,f}-bC_{\alpha,r}+mu^2}{mu} & 0 & 0 & \frac{C_{\alpha,f}}{m} & 0 & 0 \\ \frac{aC_{\alpha,f}-bC_{\alpha,r}}{Ju} & -\frac{a^2C_{\alpha,f}-b^2C_{\alpha,r}}{Ju} & 0 & 0 & \frac{aC_{\alpha,f}}{J} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & u & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{6k_d}{\tau^3} & -\frac{6Lk_d}{\tau^3} & -6\frac{k_d u+Lk_p}{\tau^3} & -\frac{6k_p}{\tau^3} & -\frac{6}{\tau^3} & -\frac{6}{\tau^2} & -\frac{3}{\tau} \end{bmatrix}$$



(a) Root Locus



(b) Maximum real part

Concept #2:

EACH HUMAN DRIVEN ROAD VEHICLE, EITHER UNDERSTEERING OR OVERSTEERING, HAS A CRITICAL SPEED DUE TO A HOPF BIFURCATION

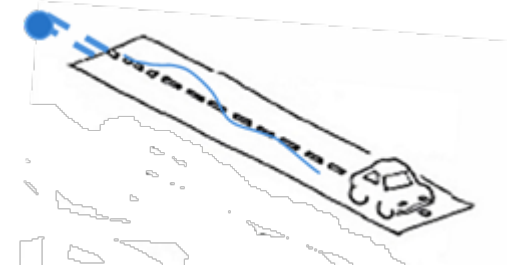
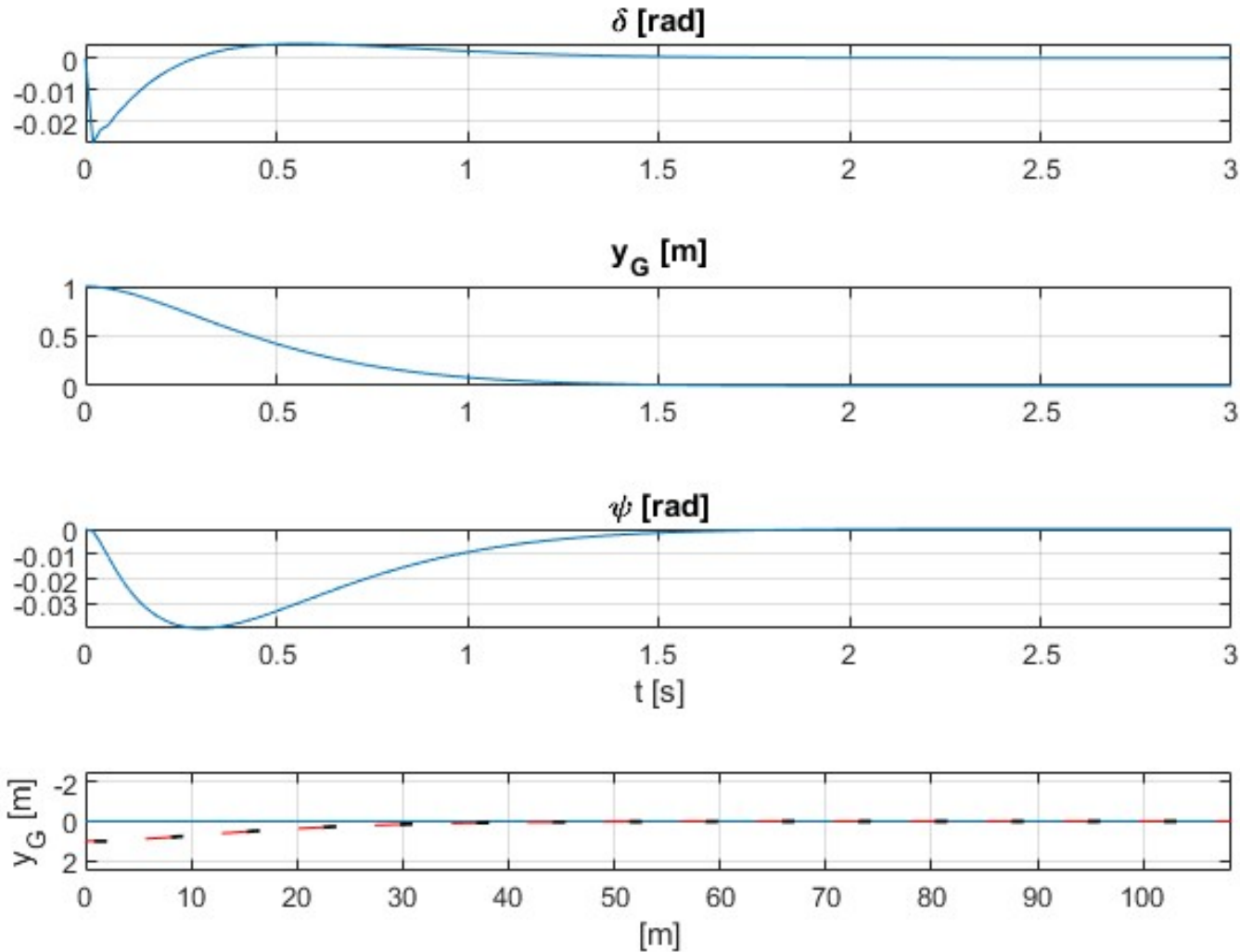


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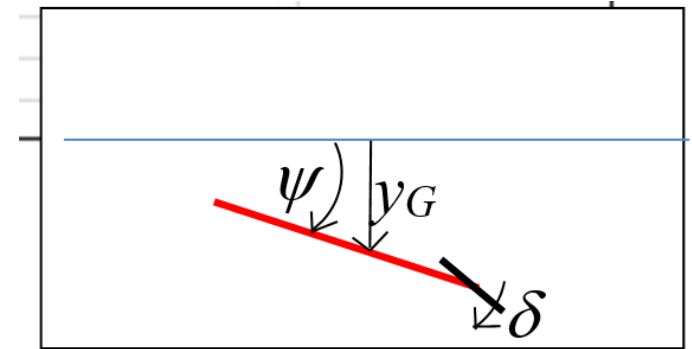
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PHENOMENON - LIMIT CYCLE



NO DELAY, NO LIMIT CYCLE



Concept #3:

THE PRIMARY CAUSE OF THE EXISTENCE OF LIMIT CYCLES IS THE DRIVER'S CONTROL DELAY



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Concept #3 (corollary):

AUTOMATED VEHICLES ARE PRONE TO LIMIT CYCLES DUE TO CONTROL DELAY



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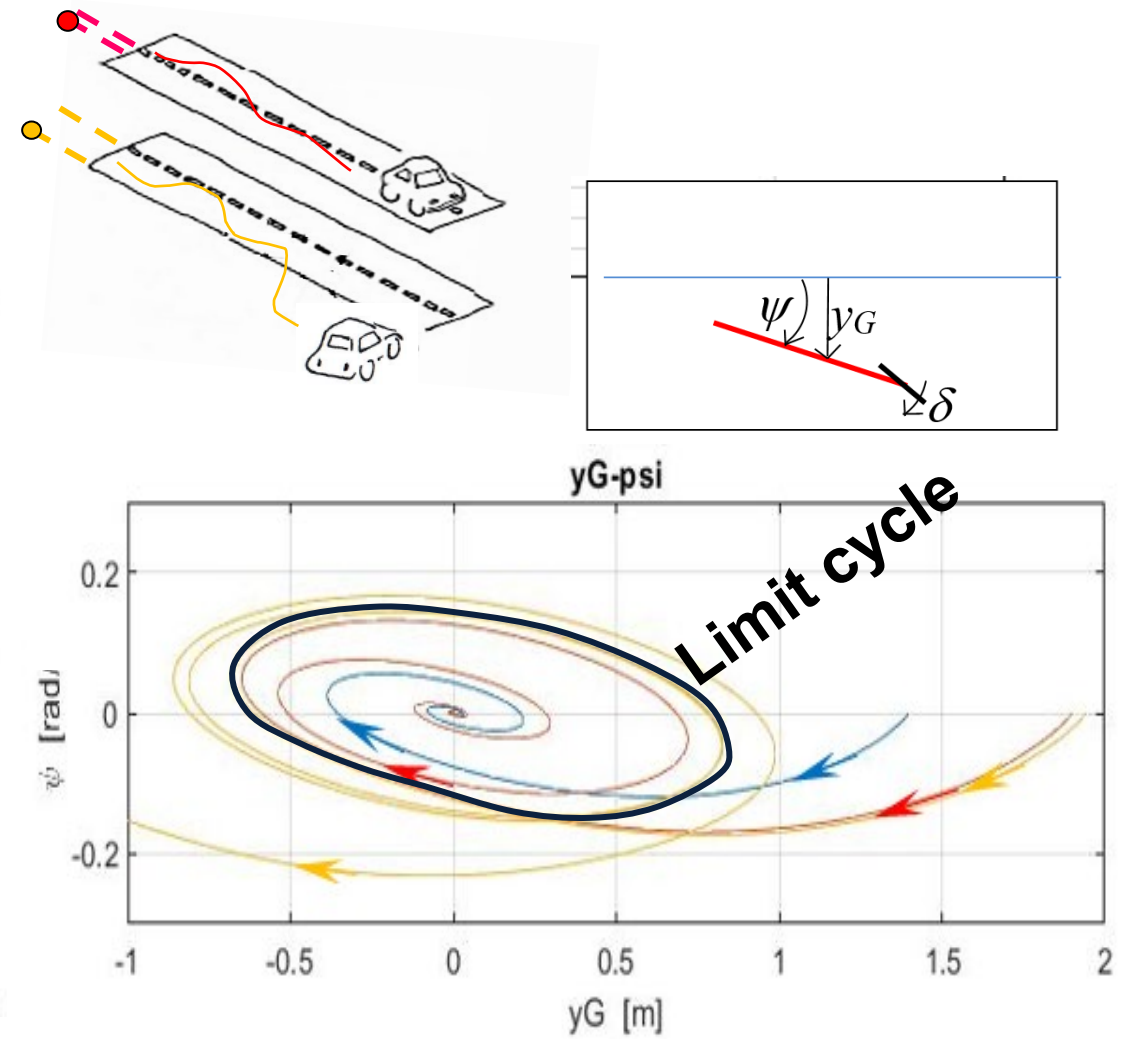
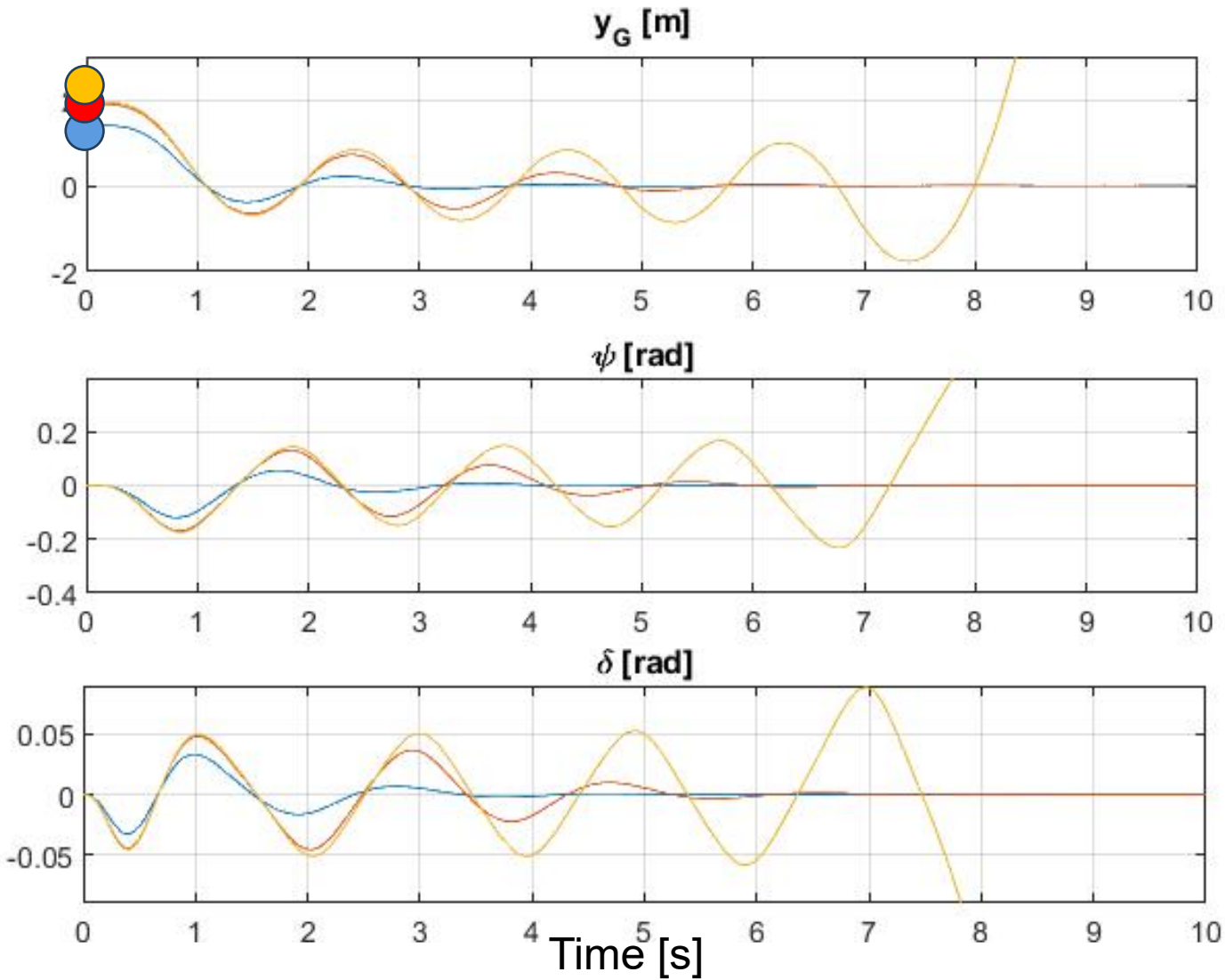
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SADDLE LIMIT CYCLE: FUNDAMENTAL CONCEPT



Concept #4:

LIMIT CYCLE IS SADDLE TYPE: IT ATTRACTS AND REPELS TRAJECTORIES



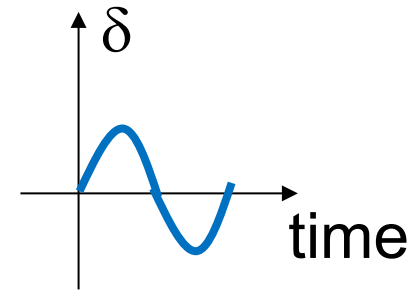
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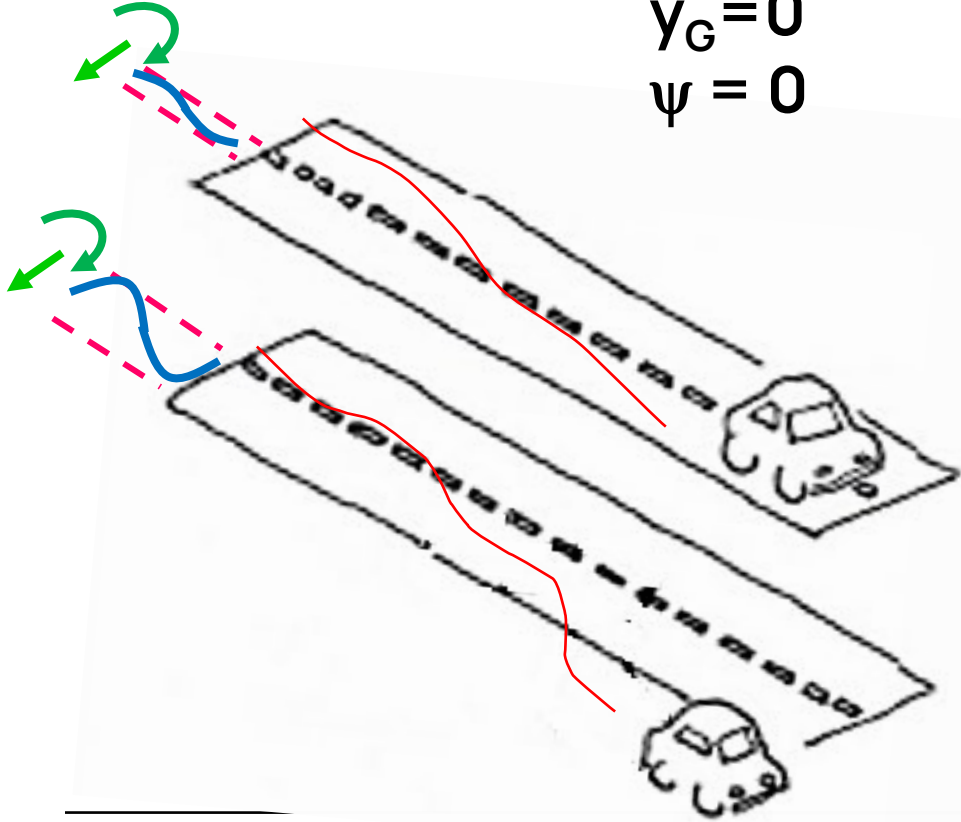


Validation: Dynamic Driving Simulator and road tests

03



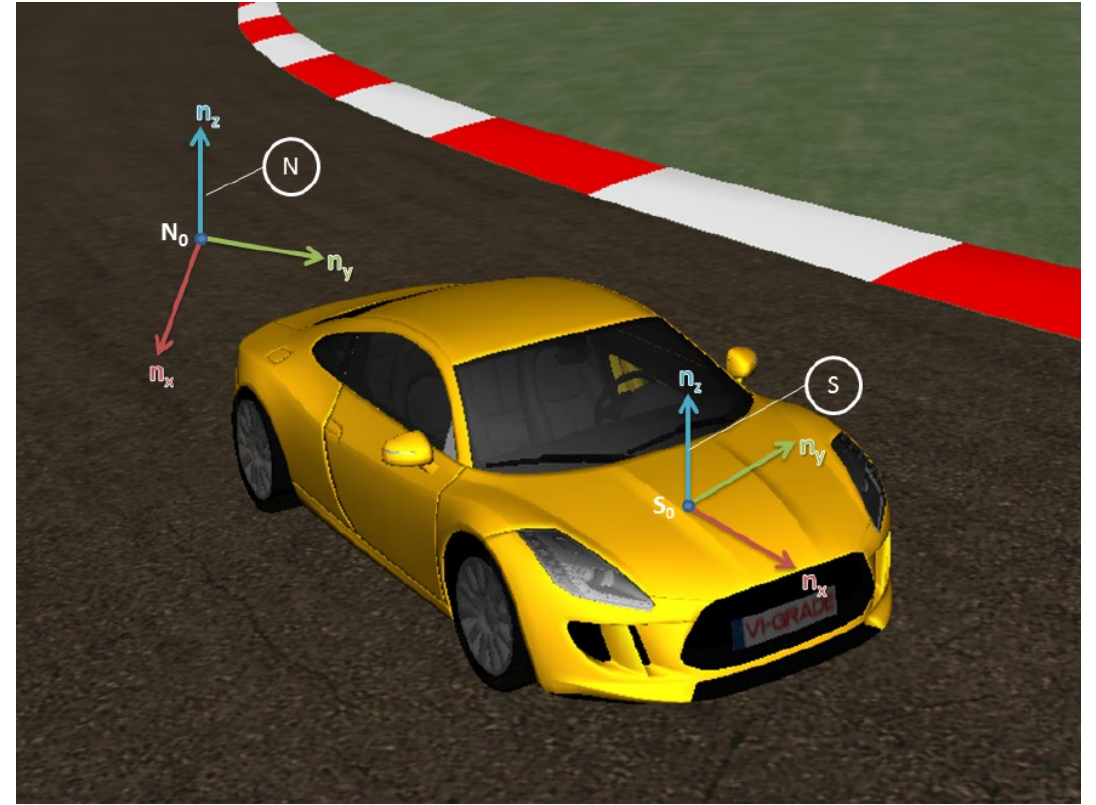
$$\begin{aligned}v &\neq 0 \\r &\neq 0 \\ \delta &= 0 \\ \gamma_G &= 0 \\ \psi &= 0\end{aligned}$$



Validation: Dynamic Driving Simulator tests



Dynamic driving simulator (Human In the Loop)

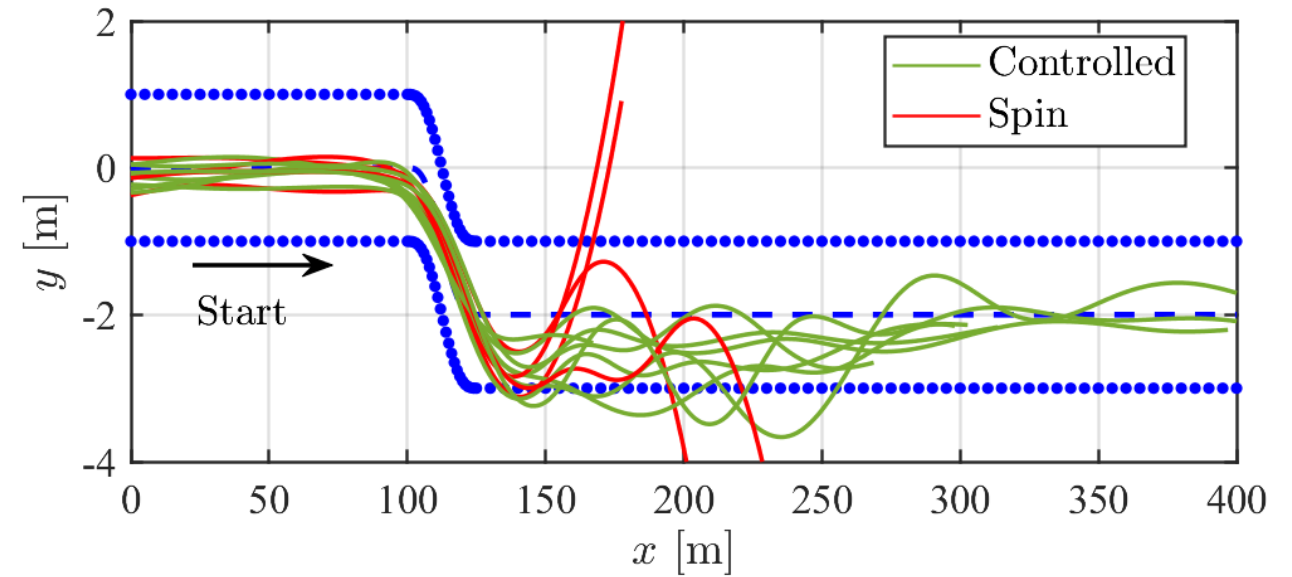


14 DOF Complete Vehicle Model (VI-CarRealTime)

Dynamic Driving Simulator tests – Lane Change

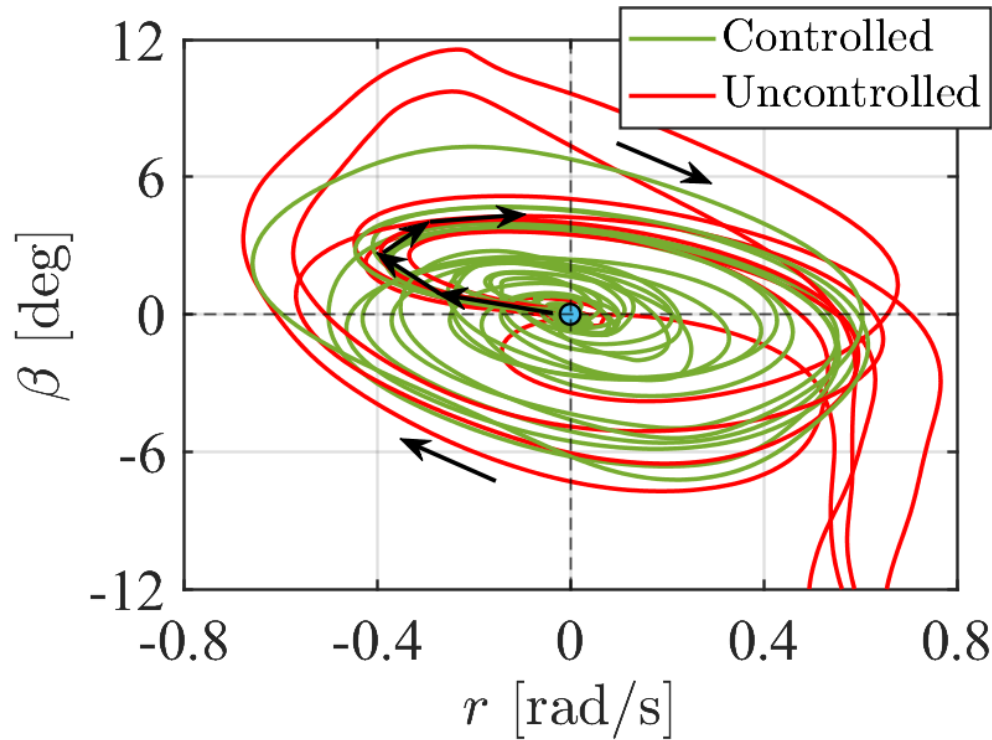
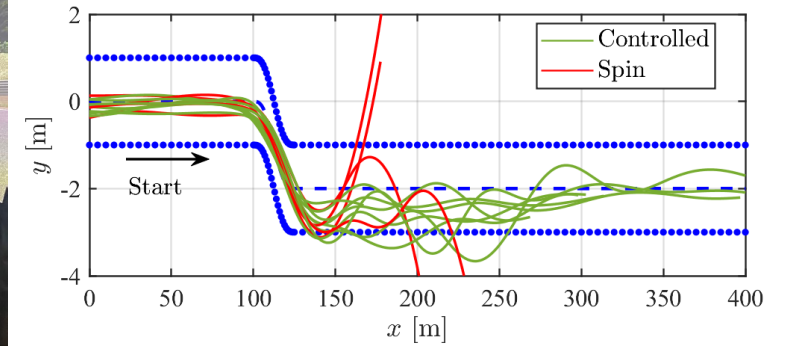


Dynamic driving simulator (Human In the Loop)

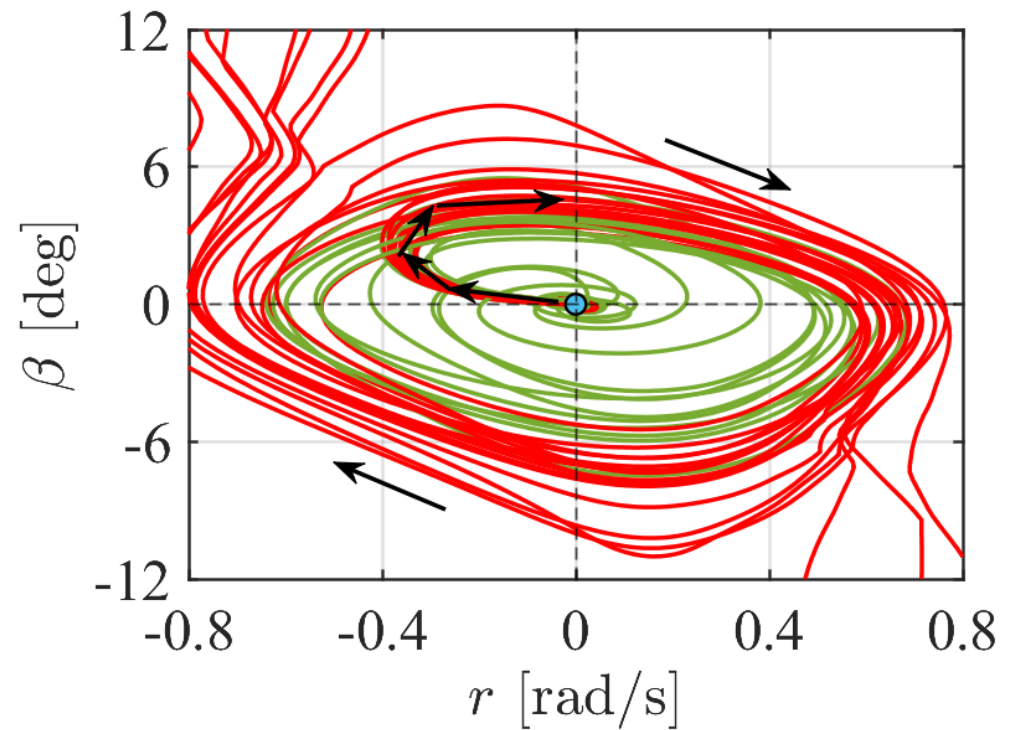


2m Lane Change

Dynamic Driving Simulator tests -

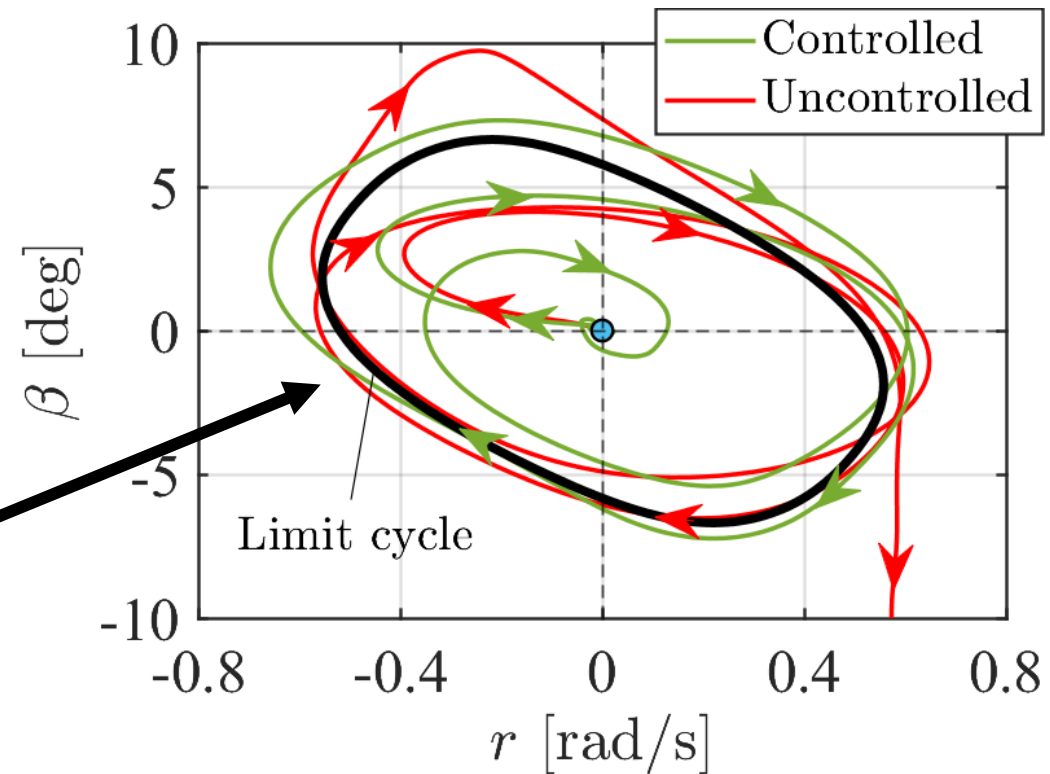
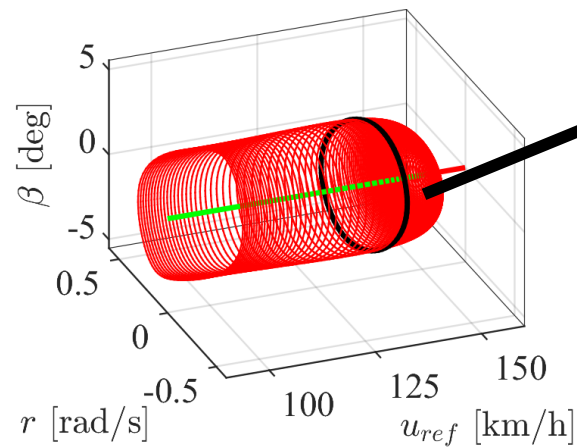


Oversteering Vehicle - 140 km/h



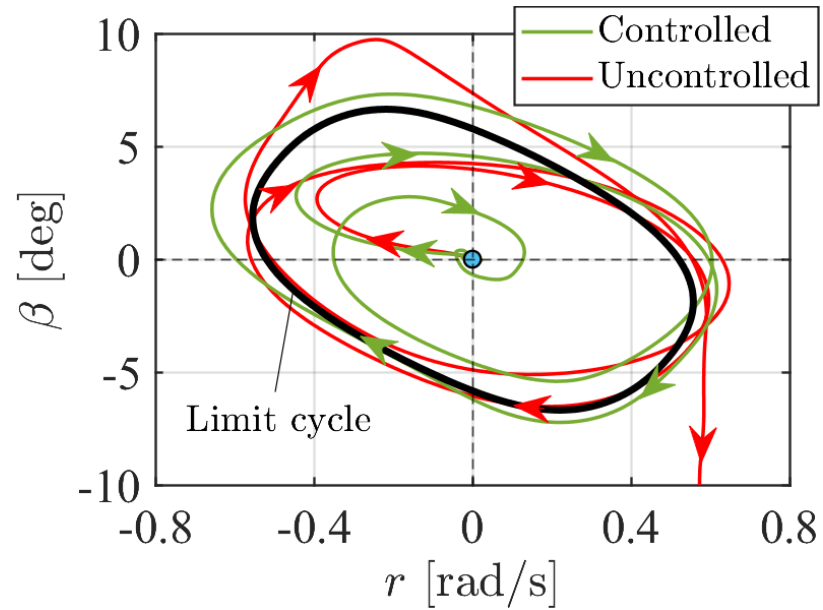
Understeering Vehicle - 220 km/h

Dynamic Driving Simulator tests – Saddle Cycle computation

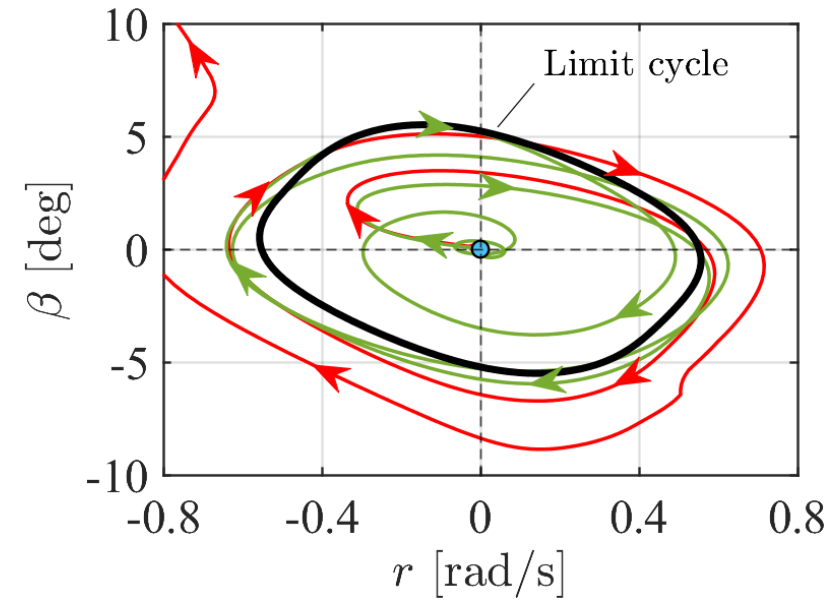


Dynamic Driving Simulator tests – Saddle Cycle computation

Oversteering Vehicle – 140 km/h

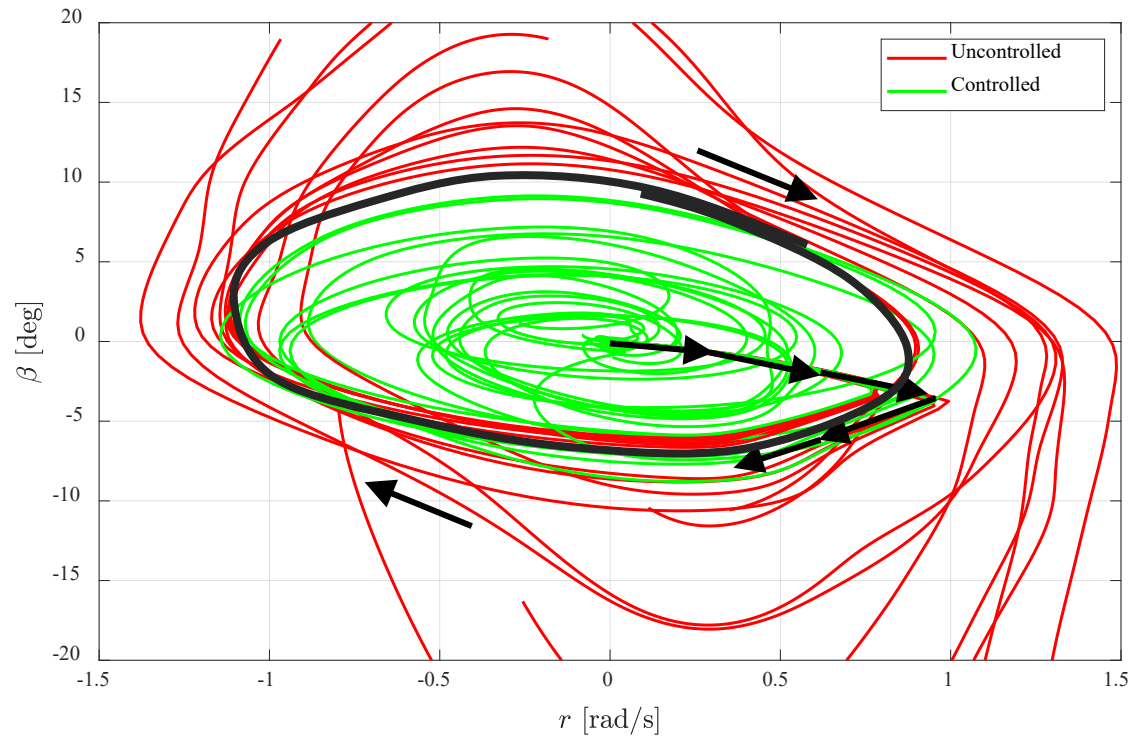


Understeering Vehicle – 220 km/h

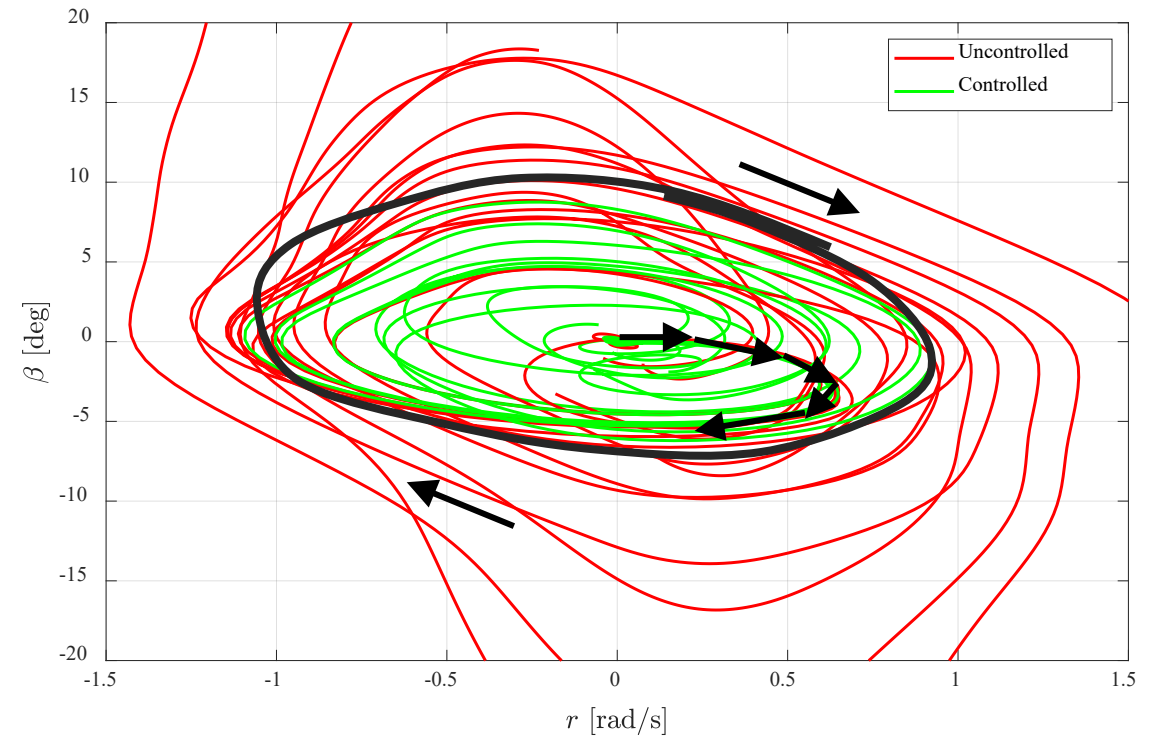


Effect of ESP On/Off on Saddle-Type Limit Cycle

150 km/h – ESP OFF - Kickplate



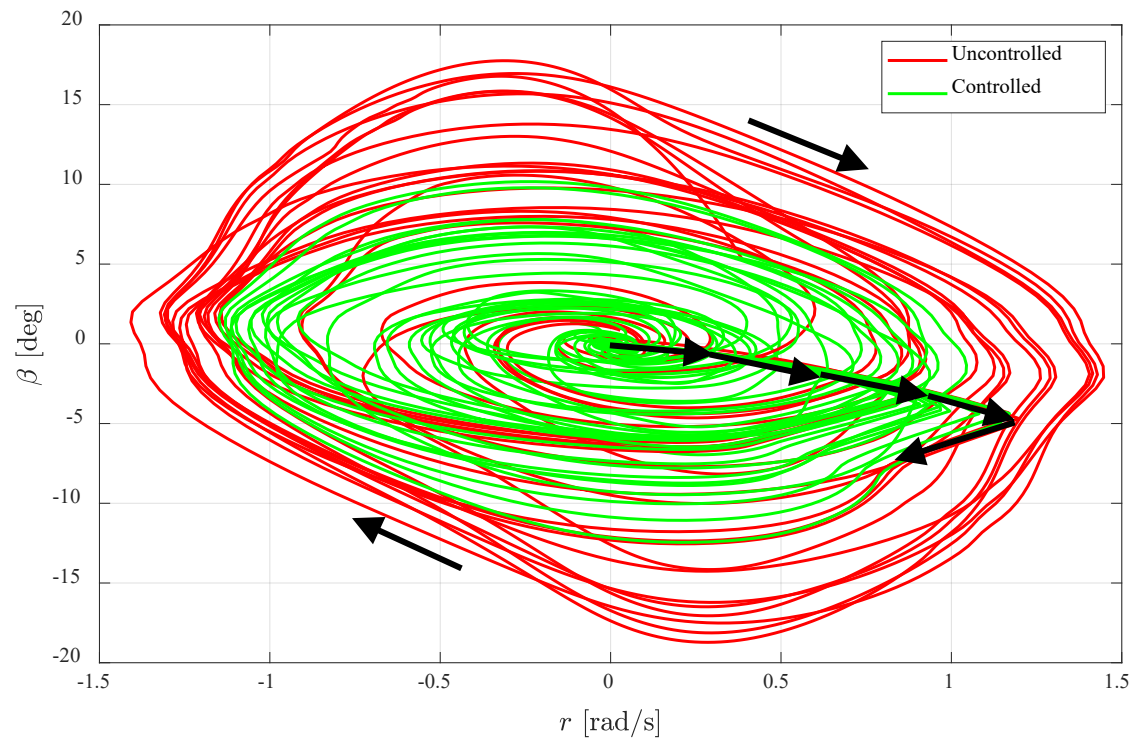
150 km/h – ESP OFF – Sinusoidal steering



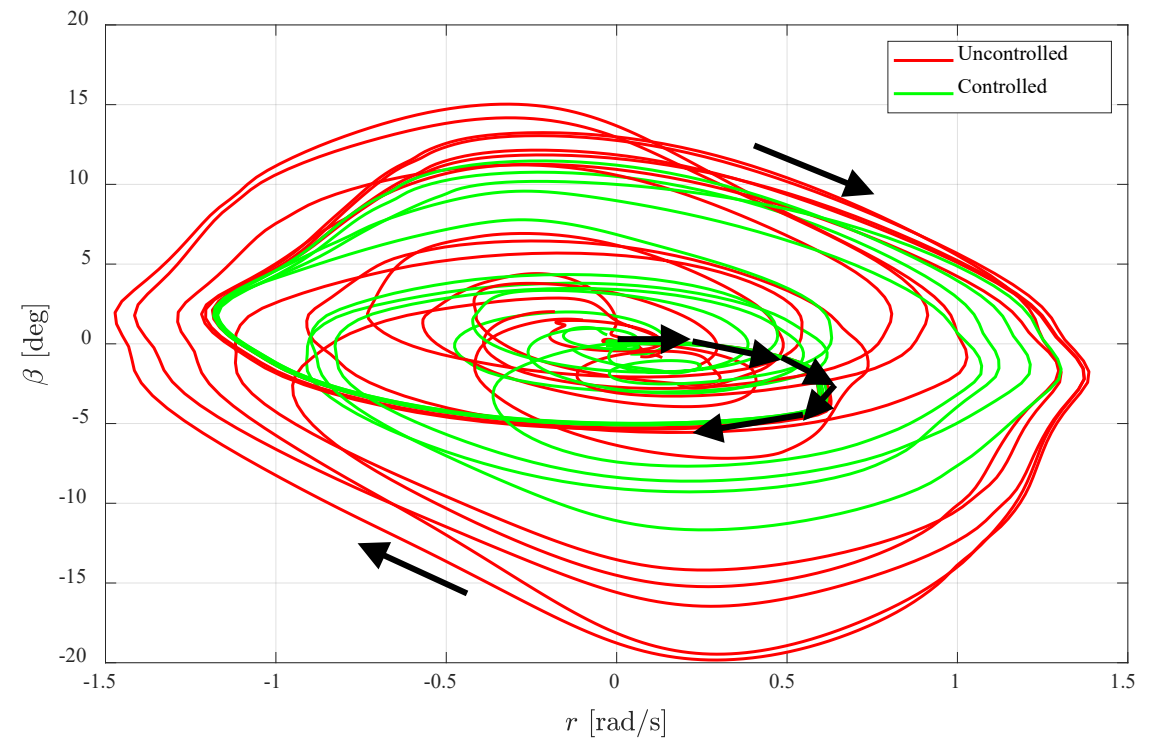
Different disturbance, same limit cycle

Effect of ESP On/Off on Saddle-Type Limit Cycle

150 km/h – ESP ON - Kickplate



150 km/h – ESP ON - Sinusoidal steering



Different disturbance, same limit cycle

safety

comfort

LIMIT CYCLES

HOW TO EXPLOIT THE KNOWLEDGE ON LIMIT CYCLES?

efficiency



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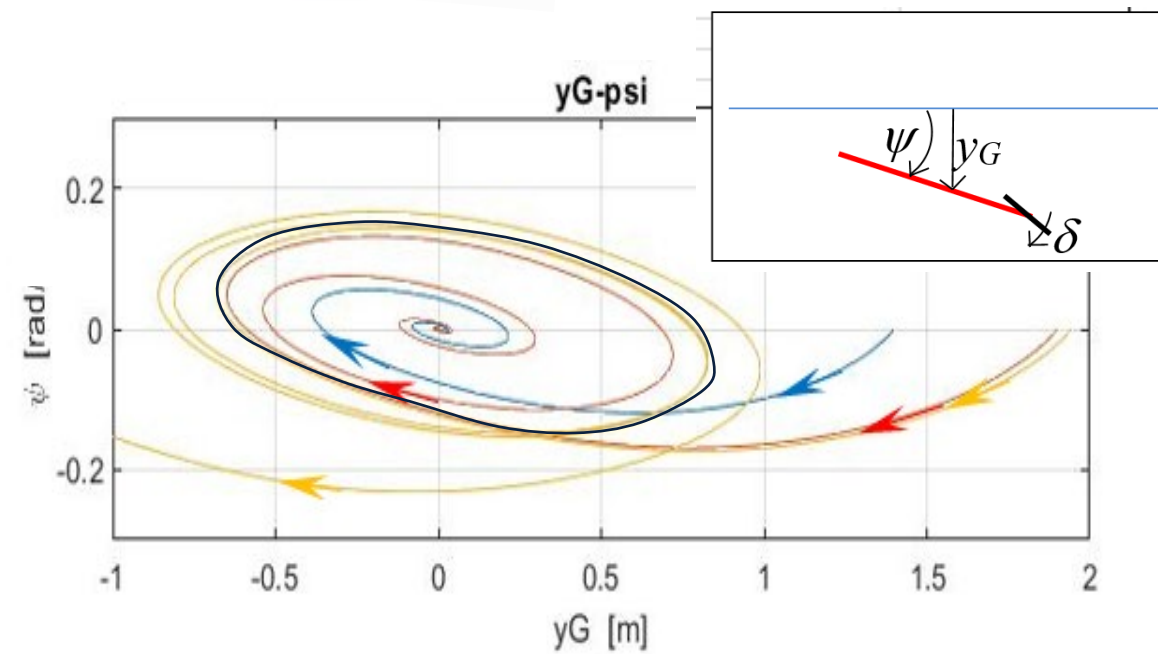
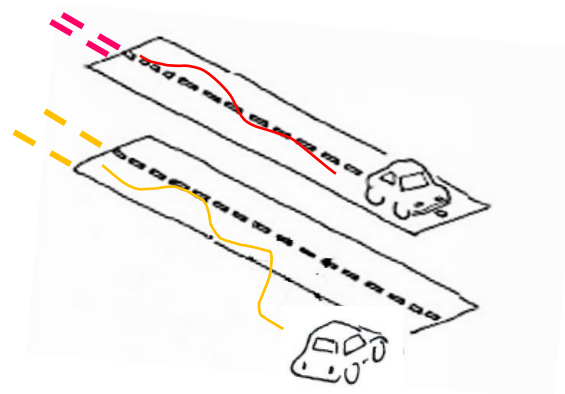
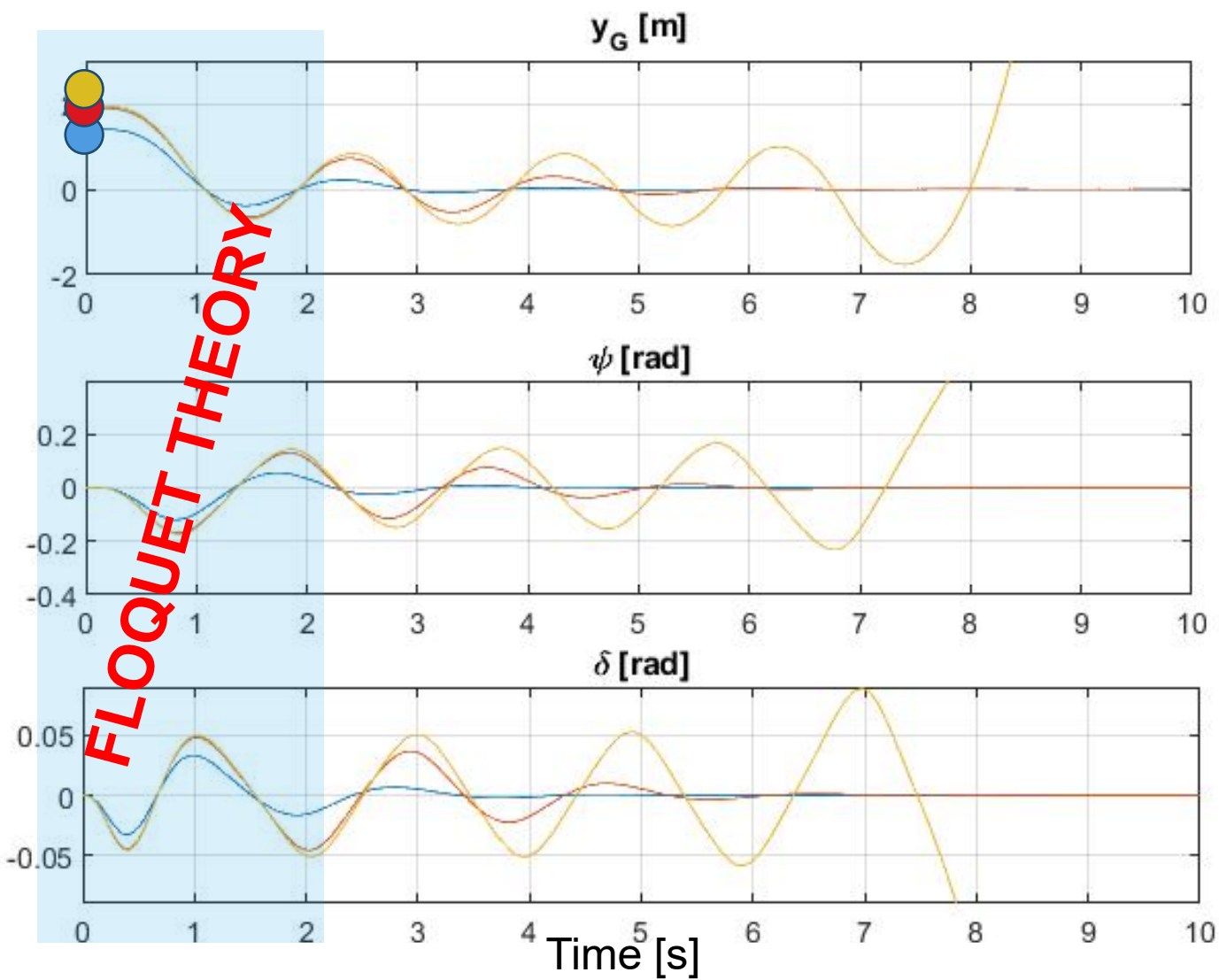
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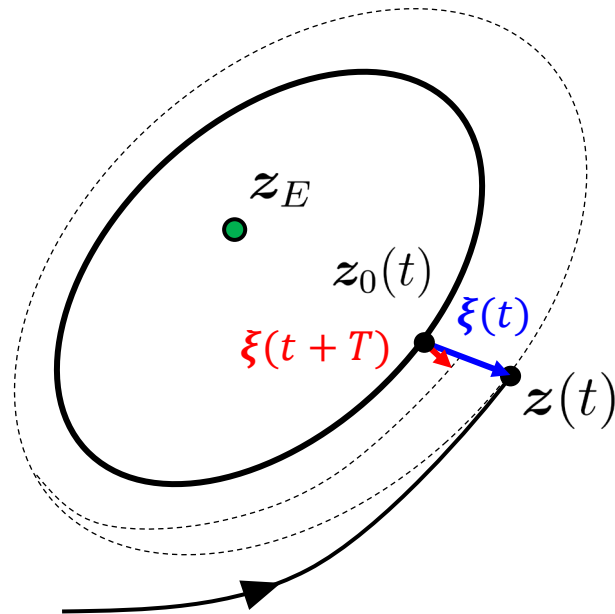
Using limit cycles to detect instability

04

EARLY DETECTION OF UNSTABLE MOTION



Detection of Instability - Analysis of orbits near the limit cycle



Linearization of EOM near the cycle

$$\dot{z} = f(z), \quad z \in \mathbb{R}^n$$

$$z(t) = z_0(t) + \xi(t)$$

Trajectory

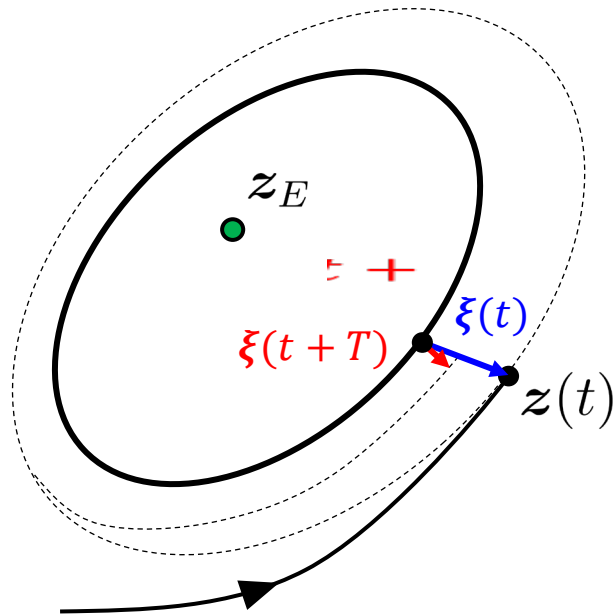
Limit Cycle

Deviation

$$\dot{\xi}(t) = \mathbf{A}(t)\xi(t) \quad \text{Variational equation}$$

$$T\text{-periodic Jacobian } \mathbf{A}(t) = \mathbf{A}(t + T)$$


Detection of Instability - Analysis of orbits near the limit cycle



Floquet Theory

$$\xi(t) = \sum_{i=1}^n c_i \xi^{(i)}(t)$$

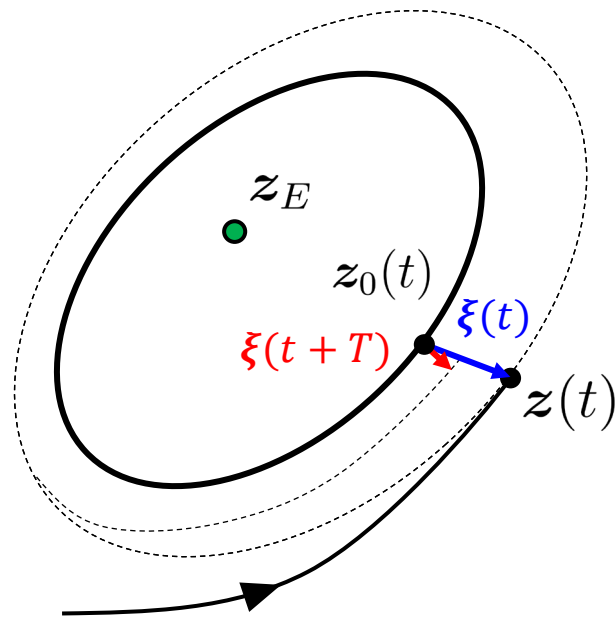
$$= c_1 \mathbf{P}(t) \mathbf{v}_1 e^{k_1 t} + c_2 \mathbf{P}(t) \mathbf{v}_2 e^{k_2 t} + \dots + c_n \mathbf{P}(t) \mathbf{v}_n e^{k_n t}$$


Floquet exponents

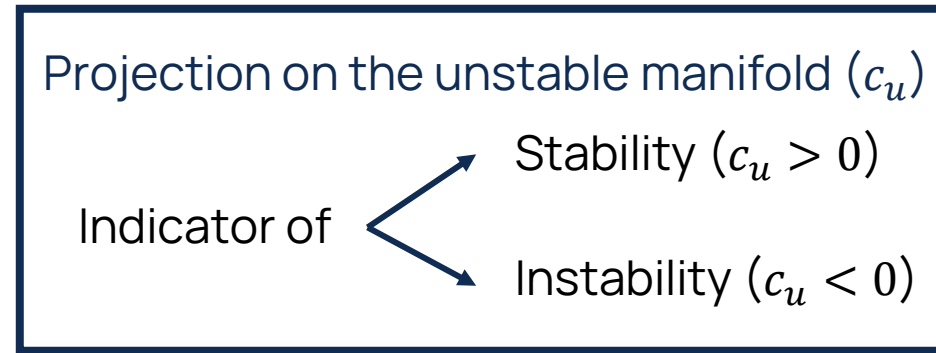
- Small **deviations** from the limit cycle decompose into **exponential modes**
- **Stable** modes: $\mathcal{R}(k_i) < 0$
- **Unstable** modes: $\mathcal{R}(k_i) > 0$
- Eigenvectors \mathbf{v}_i define **directions of growth/decay**

Vehicle-and-driver model: saddle cycle with only one unstable mode

Detection of Instability – DoS Index



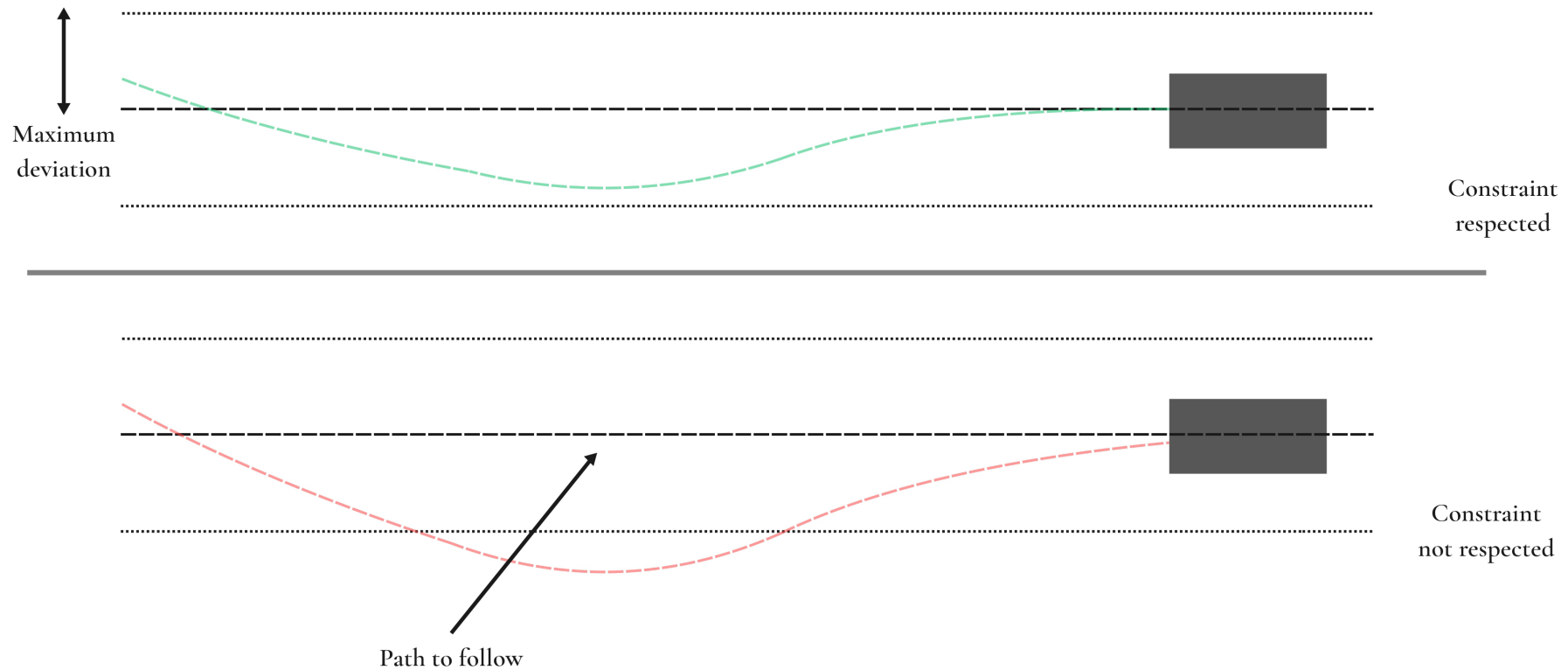
We can **predict how the motion will evolve** by evaluating the current position of the system's state **relative to the cycle's manifolds**



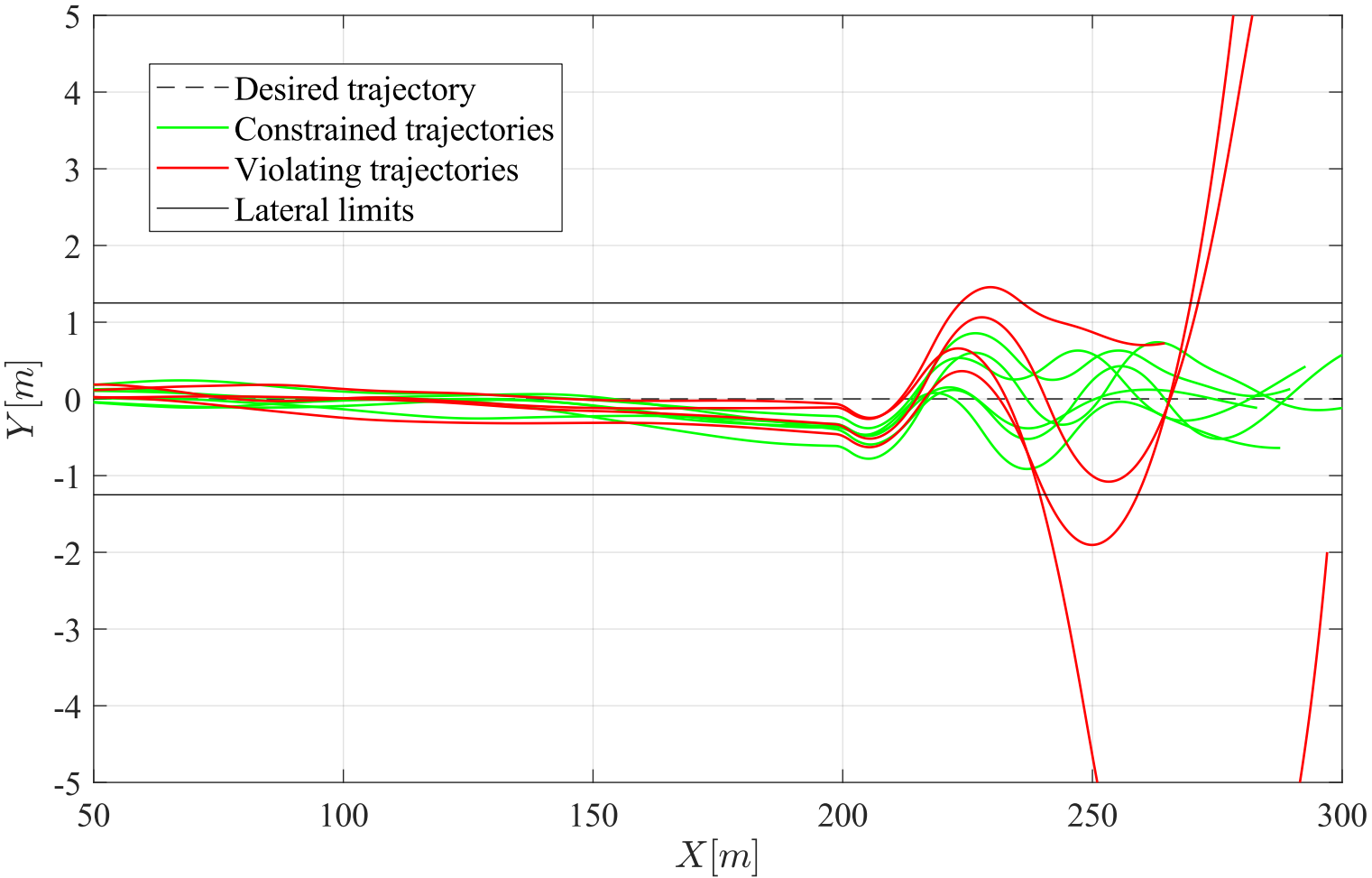
$$\xi = c_u \mathbf{v}_u + c_s \mathbf{v}_s$$

We call c_u a **Degree of Stability (DoS) index**

STABILITY + STAY IN THE LANE



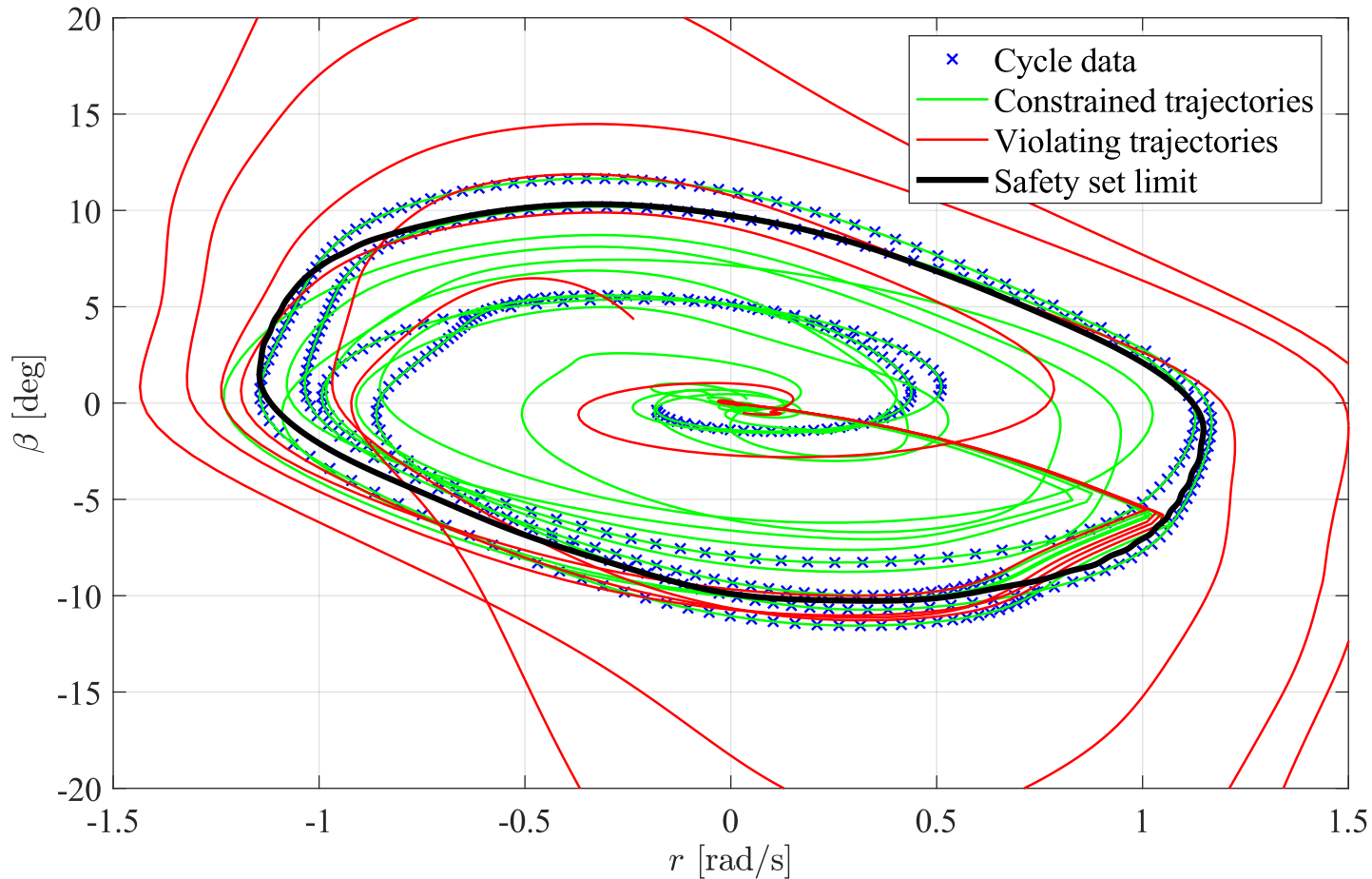
From stability to safe sets



Kick-plate disturbance

Same vehicle, using a **human driver**. Driving simulator

From stability to safe sets



Use the filtering criteria to identify a **safe set** for the system in the testes conditions.

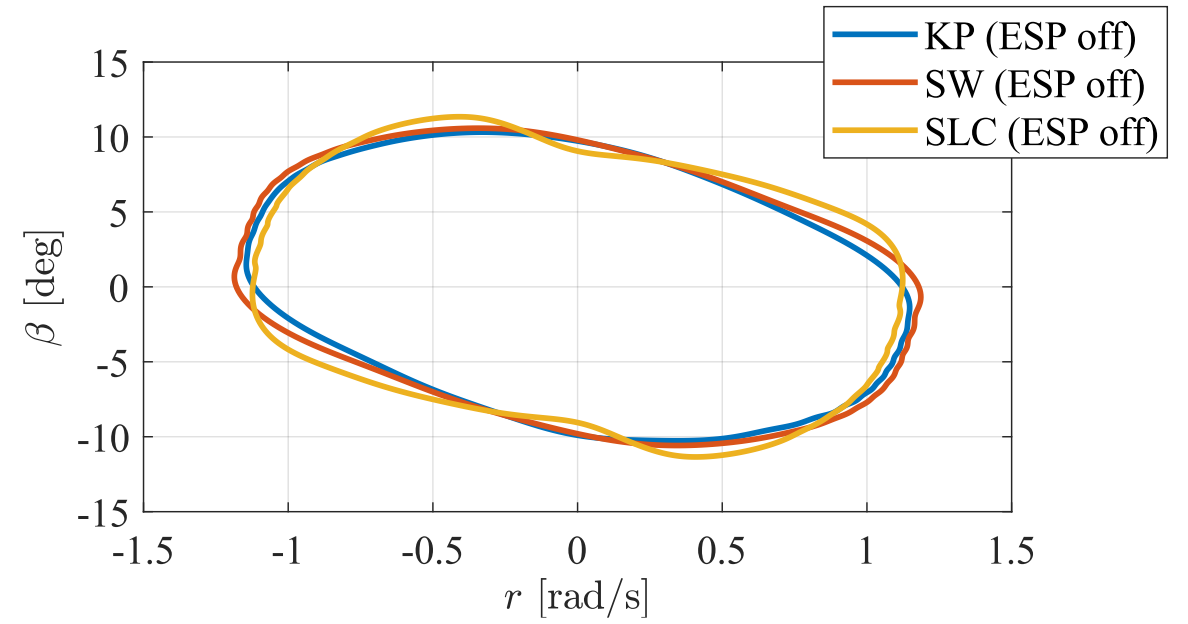
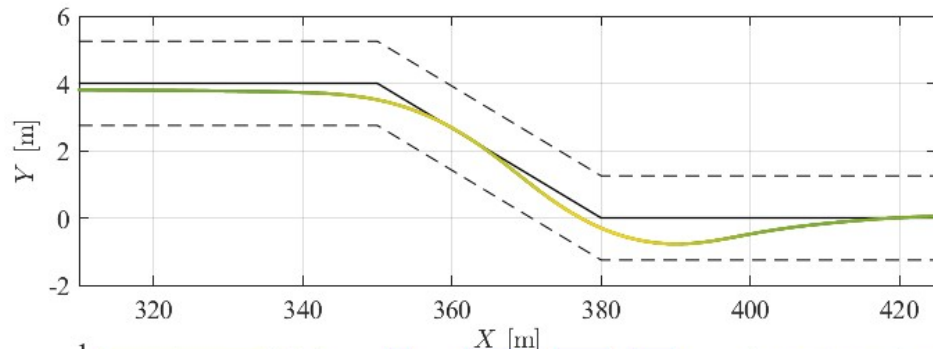
RMS minimization approach to approximate the **safety set limits**

$$f(s) = a_0 + \sum_{i=1}^m \left(a_i \cos \frac{i(2\pi s)}{s_{tot}} + b_i \sin \frac{i(2\pi s)}{s_{tot}} \right)$$

Sum of sinusoidal to approximate with a function the Safe set in the 4 considered system's states:

- β , sideslip angle
- r , yaw angle rate
- d , steering angle
- f , roll angle

From stability to safe sets



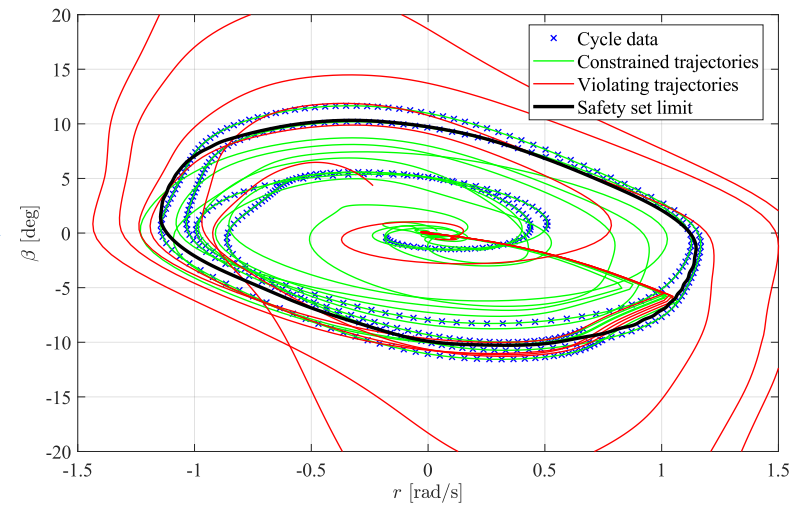
Road trajectory

Phase plane trajectory

Model-less Safety Index

Considerations on safety cycle

Model-less approach



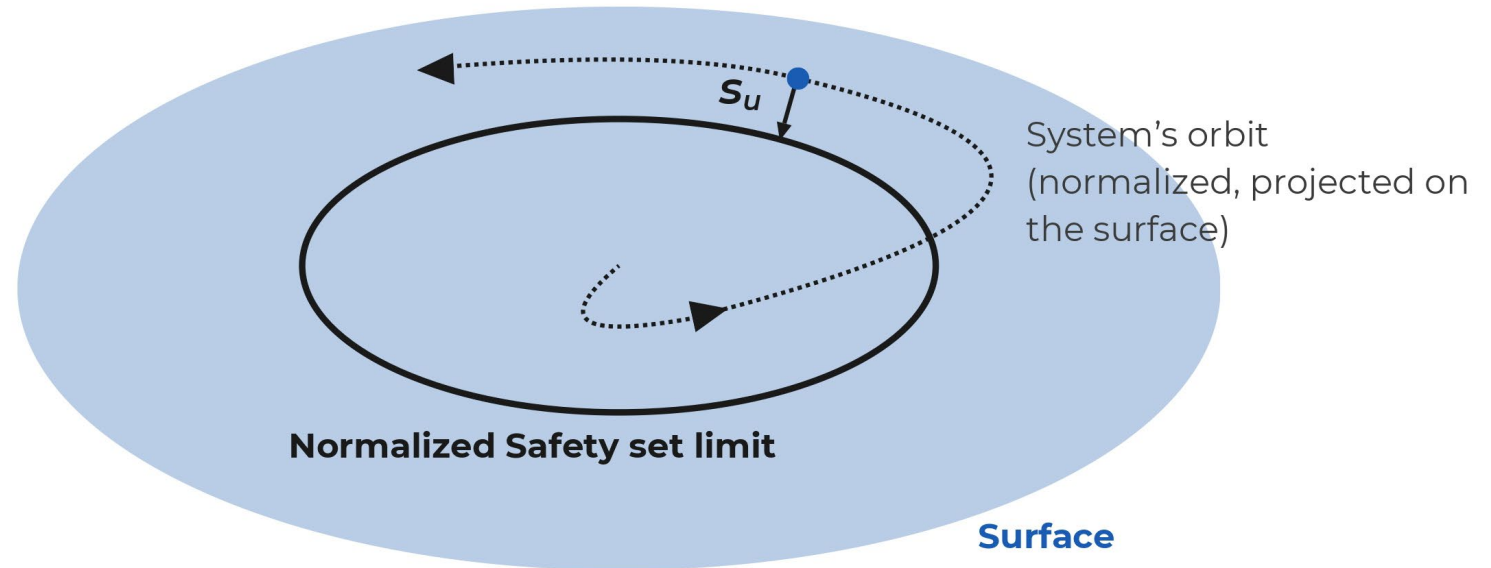
Radius / velocity	R=200 m	...	R=inf
100 km/h	SS ₁₁	SS _{i1}	SS _{n1}
...	SS _{1j}	SS _{ij}	SS _{nj}
250 km/h	SS _{1m}	SS _{im}	SS _{nm}

Interpolation to obtain an approximated safety cycle in every condition

Model-less Safety Index

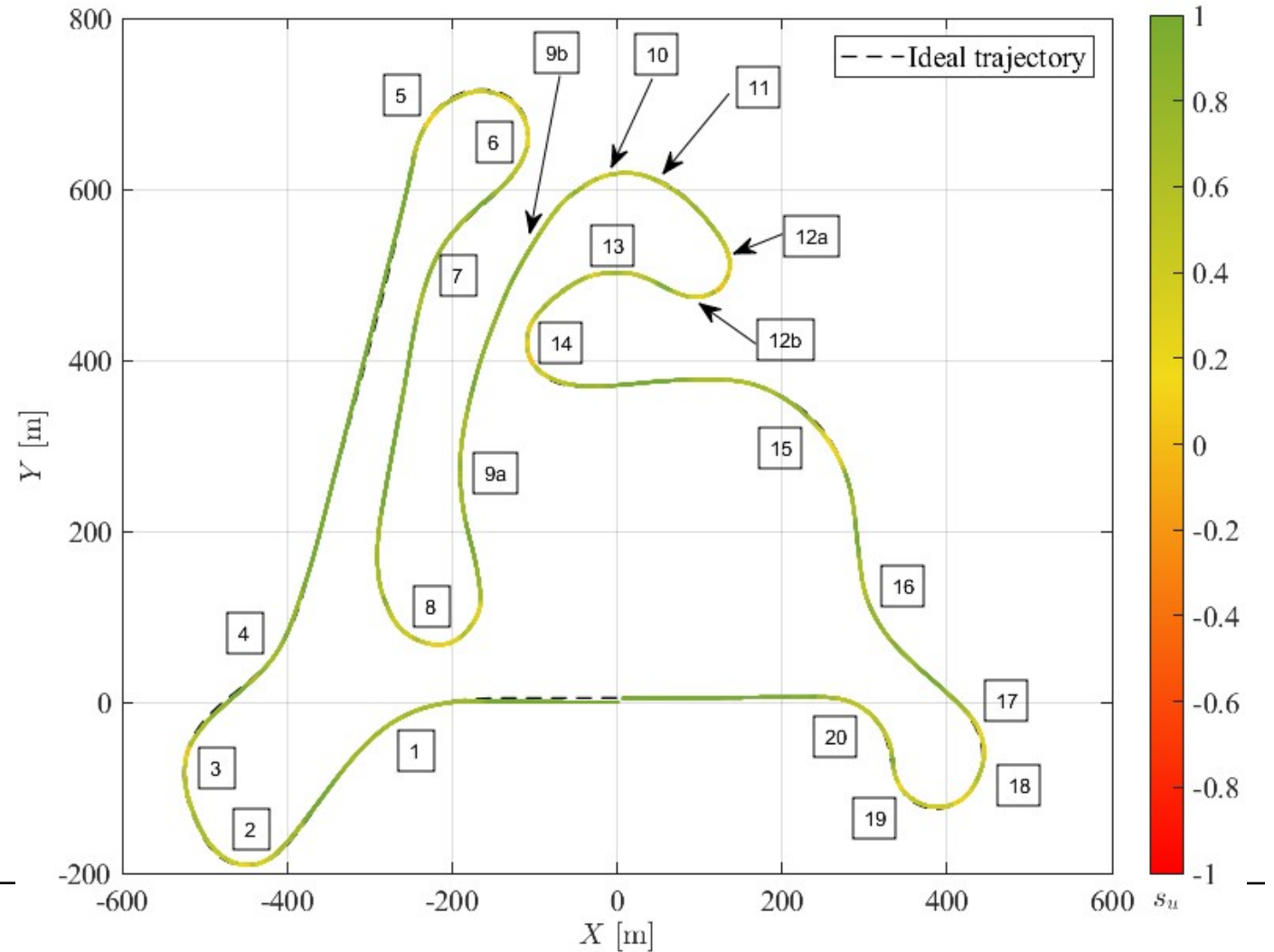
Definition and computation

1. **Normalization** of the safe set limit in $[-1,1]$ for every state
2. Computation of the **surface** passing through the normalized safe set
3. **Projection** of the normalized System's orbit onto the surface
4. Computation of the **minimum distance** ($|s_u|$) between the analyzed point (projected) of the orbit and the safe set limit
5. Computation of the **sign of the safety index** (positive if inside the Safety cycle)



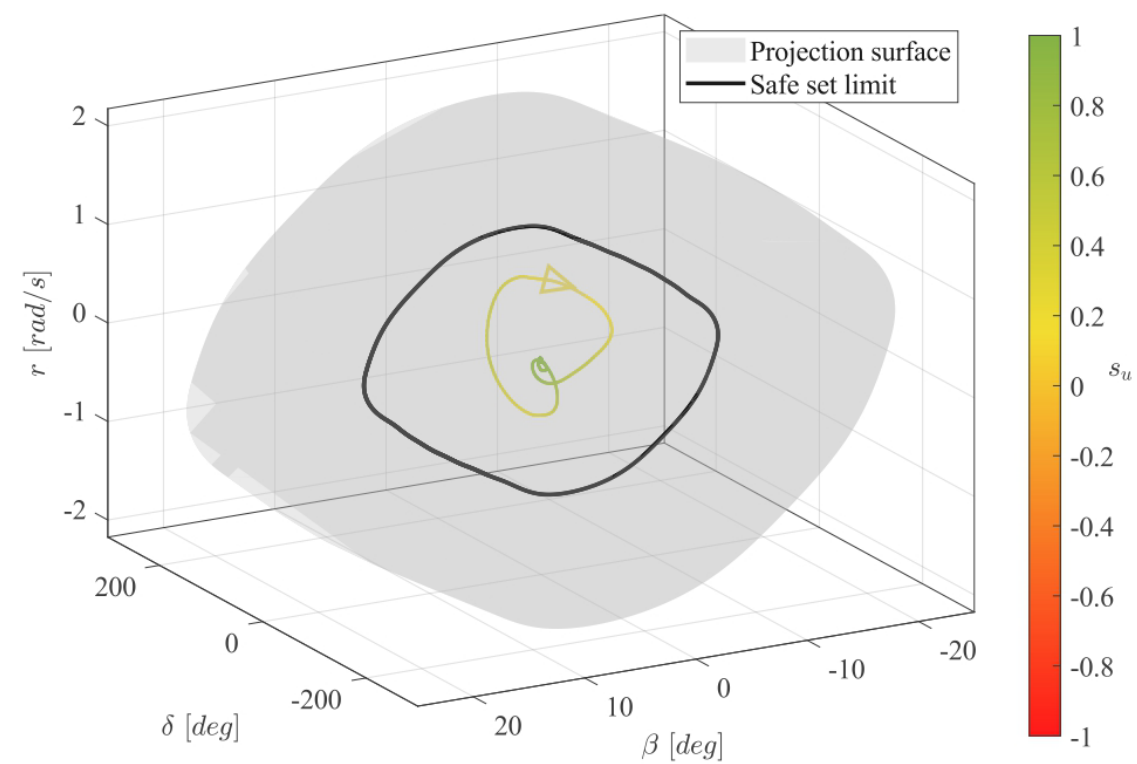
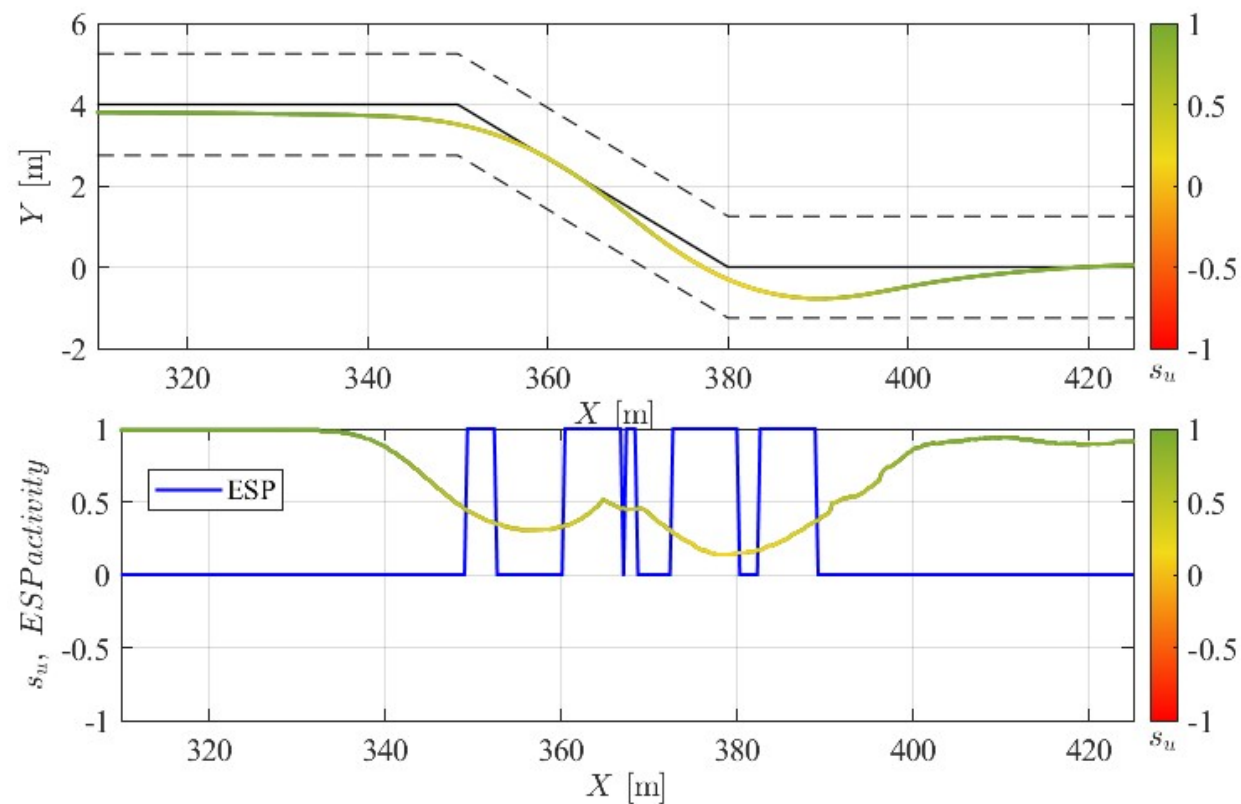
Safety index for Motorsport

Track application, non-violating manouver



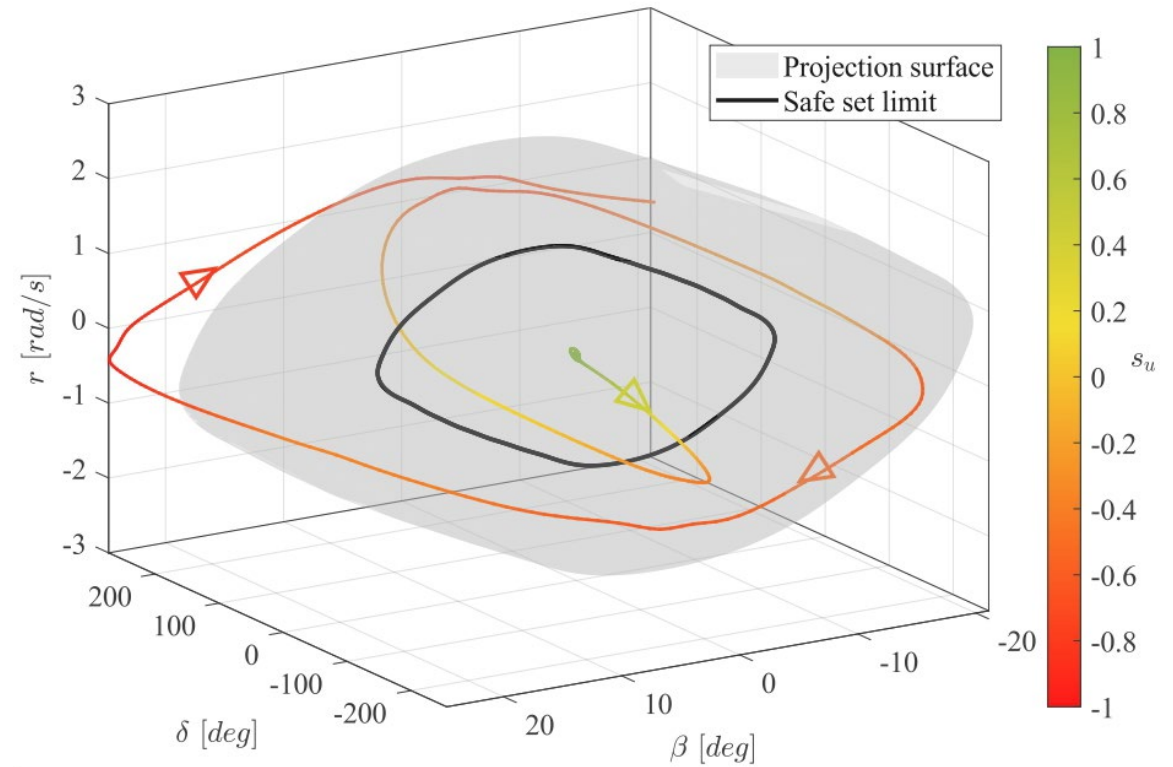
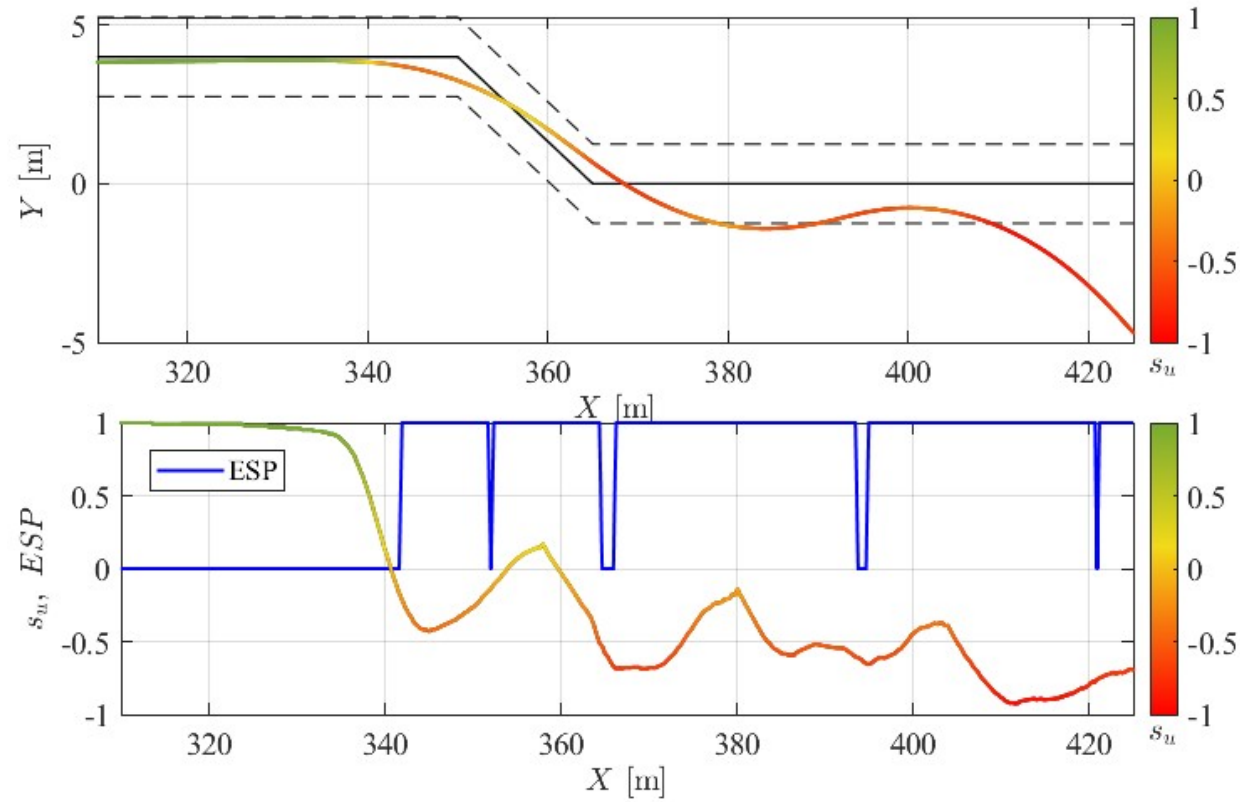
Safety index and ESP

Single Lane Change, non-violating manouver



Safety index and ESP

Single Lane Change, violating manouver



Concept #5:

LIMIT CYCLE CONCEPT MAY BE USED FOR

- EARLY DETECTION OF INSTABILITY
- DEFINITION OF TRAJECTORIES FOR EVASIVE MANOEUVRES
- MOTORSPORT
- IMPROVING ESP



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Conclusions

06

Conclusions

- 1. LIMIT CYCLES DO EXIST AND ARE FUNDAMENTAL TO UNDERSTAND ROAD VEHICLE TRANSIENT MOTION AFTER A DISTURBANCE**
- 2. LIMIT CYCLE CONCEPT MAY BE USED FOR IMPROVING THE ACTIVE SAFETY OF VEHICLES**

Thank you for your attention

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