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Summary

A New Stability Paradigm for Road Vehicles: From ISO 8855 (Vehicle-Centric) to Comprehensive Vehicle-and-Driver System Dynamics (Human Centric).

The Global Stability of Road Vehicle Motion (STAVE) project aims to fundamentally redefine the paradigm of vehicle stability, shifting focus from classical, linear approaches to a rigorous framework based on nonlinear dynamics and the dynamics of the complete road system, which is crucial for advancing active safety and the development of Connected and Automated Vehicles (CAVs). Our analysis highlights two major shortcomings in the current normative concepts, particularly within the ISO 8855 vocabulary:

1. The Stability Definition is Limited to the Vehicle, Excluding the Vehicle-and-Driver System: Traditional vehicle dynamics studies often analyze the stability of the bare vehicle—often using linearized models—and rarely introduce the driver into the control loop. However, vehicle stability inherently deals with the stability of the combined **vehicle-and-driver system** (whether the driver is human or non-human/artificial). Research has demonstrated that the driver's steering action can generate unstable motion even in vehicles that are otherwise stable under fixed control, such as understeering vehicles. Furthermore, the current ISO 8855 vocabulary **contains scientifically flawed expressions** like "oscillatory stability" and "non-oscillatory instability," as it is the *motion* that is oscillatory, not the stability itself. STAVE aims to align the technical concepts used in vehicle dynamics with the rigorous theoretical concepts of Systems Theory by incorporating the full vehicle-and-driver interaction.

2. Stability Definitions are Local, Lacking the Concept of a Basin of Attraction for Nonlinear Systems: The definitions in the current standard rely on examining the vehicle's response to a "small temporary disturbance input" near a prescribed steady state. This approach stems from the mathematical models that often engineers use to study the stability of mechanical/physical systems. The linearization approach of nonlinear equations of motion of a mechanical system **is inadequate for studying real-world scenarios** involving large disturbances. For example, panic manoeuvres, where full nonlinear vehicle-and-driver models must be considered are related to large disturbances. Often an ambiguity is implied, since the magnitude of the "perturbation" is not precisely defined. To overcome this, STAVE employs Bifurcation Theory, a branch of Systems Theory, to characterize the system's stability globally. This methodology allows researchers to rigorously compute the **domain of attraction** (or stable domain), which mathematically defines the *threshold*—**the exact set of disturbances—that the vehicle-and-driver system can tolerate before losing control**. This quantifies the safety limit in a way that local definitions cannot.

In this final report for the STAVE (Global stability of road vehicle motion) project we will document the critical shift from local, vehicle-centric stability concepts to a global framework based on the complete vehicle-and-driver system. This report documents our findings and is structured as follows:

1. **A Critical Review of Current Normative Frameworks:** We analyze the limitations of existing standards, particularly focusing on the deficiencies in the vocabulary and local approach used by ISO 8855.

2. **Introducing New Stability Paradigms:** We present the theoretical and experimental results supporting the analysis of the vehicle-and-driver system as an integrated, nonlinear entity, using Bifurcation Theory to quantify stability globally.

3. **Normative Impact and Proposed Revisions:** We finally discuss the practical implications of our scientific results and propose actionable inputs for international standards and regulations, such as ISO 8855.

Review of the current normative framework

The foundational terminology for road vehicle dynamics, particularly concerning stability, is established in the international standard **BS ISO 8855:2011,** ***Road vehicles — Vehicle dynamics and road-holding ability — Vocabulary***. The current framework defines stability characteristics based on the vehicle's response to inputs relative to a defined *steady state*. For sake of completeness, we here report the principal definitions, mainly reported in Chapter 12 of the document.

Equilibrium and Stability (ISO 8855, Clause 12.2)

The standard grounds its stability definitions upon the concept of a "steady state":

12.2.1 steady state "state of a vehicle where the sum of the applied external forces and moments and the inertial forces and moments which balance them form an unchanging force and moment system over an arbitrarily long time period".

Stability is then assessed by considering the vehicle's motion following a *small temporary input,* either a *control input* (an input voluntary given with the intent of maintaining or inducing a change in the motion of the vehicle, see 11.1.1) or a *disturbance input* (all the other inputs, see 11.1.2).

The types of stability characteristics that are defined are based on this local assessment. They are classified by whether the motion returns to the original steady state or whether the motion exhibits oscillations:

12.2.3 non-oscillatory stability "stability characteristic at a prescribed steady state (12.2.1) if, following any small temporary disturbance input (11.1.2) or control input (11.1.1), the vehicle returns to the steady state without oscillation".

12.2.4 non-oscillatory instability "stability characteristic at a prescribed steady state (12.2.1) if a small temporary disturbance input (11.1.2) or control input (11.1.1) causes an ever-increasing vehicle response (12.1.1) without oscillation".

12.2.6 oscillatory stability "stability characteristic at a prescribed steady state (12.2.1) if a small temporary disturbance input (11.1.2) or control input (11.1.1) causes an oscillatory vehicle response (12.1.1) of decreasing amplitude and a return to the original steady state".

12.2.7 oscillatory instability "stability characteristic at a prescribed steady state (12.2.1) if a small temporary disturbance input (11.1.2) or control input (11.1.1) causes an oscillatory vehicle response (12.1.1) of ever-increasing amplitude about the initial steady state".

12.2.5 neutral stability "stability characteristic at a prescribed steady state (12.2.1) if, as a result of any small temporary disturbance input (11.1.2) or control input (11.1.1), the vehicle attains a new steady state".

The standard also offers clarifying comments in Annex A, noting that the steer properties and stability characteristics depend upon test conditions and operating conditions, and may change based on the magnitude of the input, as the system is nonlinear. For instance, **non-oscillatory instability** may be illustrated by operating an oversteer vehicle above the critical speed, leading to motion of ever-decreasing radius unless the driver compensates. Conversely, **oscillatory instability** can be illustrated by the free control response after a pulse input, potentially resulting in the vehicle "spinning out".

Discussion

We now critically evaluate the conceptual current framework of stability within current ISO 8855 standard. We will put emphasis on four conceptual problems.

PROBLEM 1: THE LIMITATION OF VEHICLE-CENTRIC STABILITY DEFINITIONS

Current stability definitions in ISO 8855 address vehicle stability based on the isolated vehicle, decoupled from the control action of the human or non-human driver. Although it may be understandable that a standard primarily targeting vehicle manufacturers adopts this limited perspective, excluding the driver disregards the system's fundamental operational purpose.

Stability inherently deals with the stability of the entire system composed of the vehicle and the driver (human or not). Restricting the analysis solely to the vehicle model makes the stability assessment prone to a lack of realism, particularly because the interaction between the vehicle and the driver can fundamentally alter the overall dynamic behaviour. The stabilizing or destabilizing influence of the driver highlights the necessity of considering the vehicle-and-driver interaction for safe operation [2].

To better understand this concept, let focus on the presented concept of Neutral Stability: as defined in the document, Neutral Stability characterizes stability at a steady state if, following a small temporary disturbance, the vehicle "attains a new steady state". While mathematically sound within a simplified model, the practical utility of analyzing this concept in a regulatory context is limited. If, following a disturbance, the car settles into a new steady state that deviates from the desired path, the system is fundamentally moving away from where it is intended to go, even if the new equilibrium state is technically stable. Analyzing this new, undesired steady state does not provide sufficient insight into whether the control system (the driver) has successfully rejected the disturbance. Stability analysis must therefore prioritize the vehicle's ability to converge back to the original, desirable equilibrium point or trajectory.

PROBLEM 2: STABILITY DEFINED BY EQUILIBRIUM VS. TRAJECTORY

A second critical issue arises from the current conceptual approach to stability within regulatory documentation, namely the tendency to exclusively associate stability properties with a **steady state** or **equilibrium** condition.

The vocabulary set forth in standards like ISO 8855 explicitly ties the definitions of stability—including non-oscillatory stability, neutral stability, oscillatory stability, and their corresponding instabilities—to a "prescribed **steady state** (12.2.1)". The steady state itself implies an "unchanging force and moment system".

However, modern nonlinear dynamics studies, such as those conducted within the STAVE project, demonstrate that the behavior of the vehicle-and-driver system cannot be accurately restricted to linear analysis around equilibrium (steady state) [3,4,7,8,9]. Considering the complex interaction and the resulting highly non-linear behavior, stability must be understood in the context of the entire trajectory, not just a fixed point or equilibrium. Bifurcation Theory, which is the proposed methodology to understand such complex non-linear systems, reveals that the vehicle's motion (without introducing the driver, so as in the context adopted in the ISO 8855 document) may be characterized by the existence of *limit cycles*, which represent non-stationary periodic solutions [1].

Crucially, stable operation is not always synonymous with the vehicle maintaining a perfectly unchanging state. Research shows that a vehicle-and-driver model can, within certain velocity ranges, exhibit **bistability**, where the system converges either to the stable equilibrium or to a **stable periodic solution** (stable limit cycle). For instance, stable periodic solutions exist for an understeering vehicle-and-driver system between 78 km/h and 177 km/h [8].

Therefore, if the goal of stability evaluation is the maintenance or recovery of the intended trajectory, restricting the definition of stability to the recovery of the original *steady state* fails to capture the full range of safe and stable operational modes, particularly those involving stable non-stationary motion. To ensure scientific rigor, regulatory concepts should align with Systems Theory, adopting a broader vocabulary where stability is defined in relation to the general **motion** or **trajectory** rather than being strictly limited to an equilibrium state.

PROBLEM 3: CONFLATION OF STABILITY CHARACTERISTICS AND MOTION MODES

A significant conceptual difficulty within ISO 8855, is the conflation of the definition of stability—a systemic characteristic—with the nature of the resulting motion (or response). The standard defines various types of stability by attaching the oscillatory characteristic directly to the stability property itself:

- **Non-oscillatory stability** (ISO 8855: 12.2.3).
- **Non-oscillatory instability** (ISO 8855: 12.2.4).
- **Oscillatory stability** (ISO 8855: 12.2.6).
- **Oscillatory instability** (ISO 8855: 12.2.7).

From a strict Systems Theory perspective, this classification **is conceptually flawed, as it is only the *motion* (or the response of the vehicle) that can accurately be described as oscillatory or non-oscillatory**. The stability property itself should be defined independently of the oscillatory nature of the transient response.

Moreover, the utility of regulating concepts like **Non-oscillatory instability** (ISO 8855: 12.2.4) is questionable within the operational domain of a controlled vehicle (i.e., a vehicle-and-driver system). Non-oscillatory instability is typically illustrated by the dynamics of an oversteer vehicle running above its critical speed without driver compensation, where any steering input leads to a turn of ever decreasing radius. This behavior suggests that a disturbance causes an ever-increasing vehicle response without oscillation. However, when considering the comprehensive nonlinear system involving a driver (human or non-human), the mechanism of stability loss fundamentally changes:

1. **Driver Action Leads to Oscillation:** When a driver is introduced into the loop, the delay in the steering action is identified as the primary cause for the generation of limit cycles. The driver's reaction is inherently late, leading to an excess or lack of control action, which produces the periodic or oscillatory behavior characteristic of a limit cycle [5,6].

2. **Hopf Bifurcation Dominance:** The instability found in the vehicle-and-driver system, both for understeering and oversteering configurations at sufficiently high speeds, is consistently characterized by a **Hopf bifurcation**. This bifurcation leads immediately to oscillatory motion (limit cycles), which means the instability mechanism is oscillatory in nature (Oscillatory instability, ISO 8855: 12.2.7).

3. **Lack of Practical Non-Oscillatory Instability:** The research undertaken demonstrates that, due to the destabilizing/stabilizing effect of the driver's delayed input, the motion of the coupled system tends to become unstable through oscillatory behaviors (limit cycles being saddle-type cycles). If the steering delay vanishes, the Hopf bifurcation is suppressed, removing the limit cycles. Thus, the characteristic loss of stability when a driver is engaged is oscillatory, (provided that the driver does not decide to keep the steering wheel fixed – a possible strategy followed by very skilled or professional drivers).

Consequently, while mathematically possible in a fixed-control model, non-oscillatory motion is generally not the critical mechanism governing the loss of control in real-world vehicle operation where a driver is attempting to maintain a trajectory. Focusing regulatory definitions on this mode therefore offers little practical insight into safety thresholds.

PROBLEM 4: THE INSUFFICIENCY OF LOCAL STABILITY DEFINITIONS: THE NECESSITY OF GLOBAL ROBUSTNESS

A fundamental limitation of stability definitions within standard ISO 8855 is its focus on **local stability**, meaning the system response is analyzed exclusively following a "small temporary disturbance input". For instance, *Non-oscillatory Stability* (ISO 8855: 12.2.3) is defined based on the vehicle returning to a prescribed steady state after *any small temporary disturbance*. While mathematically grounded in linear system theory, restricting the analysis to infinitesimal or arbitrarily small disturbances inherently limits the practical relevance of the definition.

Vehicle safety mandates considering stability not just around a steady state, but across the entire operational domain, requiring analysis of **global stability** and the **domain of attraction** (or basin of attraction). This domain defines the set of all disturbances (thresholds) that the system can successfully absorb while maintaining a controlled motion [2-9].

The stability analysis is prone to a lack of realism if it fails to consider that any actual vehicle-and-driver is described by full nonlinear models to account for large inputs, such as emergency or "panic maneuvers," strong wind gusts, or severe road irregularities. A vehicle categorized as stable according to definition 12.2.3 might become wholly unstable if subjected to a "sensible" disturbance, such as a lateral impulse or an aggressive lane change.

The STAVE project emphasizes that the primary goal is to quantify the **threshold above which a disturbance makes the vehicle-and-driver motion unstable**. This threshold is critical because even an inherently stable vehicle (like an understeering car without a driver) can be made unstable by the driver's actions at high speeds, shrinking the basin of attraction until instability occurs via a Hopf bifurcation. Therefore, defining stability without quantifying the robustness against typical large inputs misses the crucial safety metric.

These critical issues identified in stability definitions within ISO 8855, mandate a fundamental conceptual reassessment of how vehicle stability is mentioned. Such a reassessment appears to be relevant in the context of emerging automated vehicles and advanced dynamics studies like the ones performed within STAVE.

To support the critique and demonstrate the quantitative potential of a Comprehensive Vehicle-and-Driver System Dynamics, the following section will introduce the minimum necessary mathematical framework to present the results obtained by the project. This includes establishing a vehicle-and-driver model that captures the essential nonlinear dynamics required for bifurcation analysis and the rigorous quantification of stability thresholds.

Introducing New Stability Paradigms

The analysis undertaken by the STAVE project necessitates shifting the perspective from analyzing the vehicle in isolation to considering the stability of the integral **vehicle-and-driver system**. To scientifically characterize the non-linear behaviors crucial for understanding panic maneuvers and large disturbances, the modeling approach must be based on full non-linear dynamics, exploiting Bifurcation Theory.

The STAVE approach

For conducting bifurcation analysis and deriving stability criteria, a minimal but effective mathematical framework is used, representing the state-of-the-art knowledge on vehicle–driver interaction. The results obtained through this analysis are then feedforwarded to a complex realistic vehicle model driven by a human driver, using the DRISMI facilities at Politecnico di Milano, thus allowing for the qualitative validation of the theoretical prediction and for a real quantification of the obtained stability threshold.

SIMPLE VEHICLE-AND-DRIVER MODEL

The simple vehicle-and-driver model comprises two main parts:

1. **Vehicle Dynamics (2 Degrees of Freedom, 2 DoF):** The vehicle is modeled as a single-track system (rigid beam) accounting for lateral motion v and yaw rotation r, utilizing non-linear tire characteristics

(Pacejka Magic Formula, adapted). This non-linear approach is crucial because the vehicle's non-linear behavior plays a crucial role during high slip angles reached in emergency situations.

2. **Driver Control Loop:** The driver model is employed to control the steering wheel δ based on the path error e and its derivative $\dot{e}$ computed at a predictive distance $L = T_{prev}u$. Critically, the steering action is applied with a control delayτ. Since the typical driver delay is approximately 0.2 s, this time delay is approximated using a third-order Taylor series expansion to model the delayed steering dynamics accurately. The complete system results in a set of equations governing seven state variables, including vehicle lateral speed and yaw rate, and variables related to driver steering action, position, and yaw angle.

This framework allows for rigorous analysis, through the application of Systems Theory tools, of the system's stability, which, for a given vehicle, depends on vehicle speed u and driver characteristics (control gains, delay, and preview time).

DRISMI (DRIVING SIMULATOR OF THE POLITECNICO DI MILANO)

The DriSMi (Driving Simulator of the Politecnico di Milano) is a dynamic driving simulator utilized to study vehicle stability in safe conditions, overcoming the danger associated with performing such maneuvers on an actual track. The facility is a state-of-the-art cable-driven simulator featuring a full-scale cockpit and nine degrees of freedom.

Key features and components of the DriSMi facilities include:

• **Motion Platform:** A base platform is employed for generating low-frequency longitudinal, lateral, and yaw motions (up to 3 Hz, within a 4x4 m working space, and allowing yaw rotation). The cable actuation system allows for high yaw angles.

• **Cockpit Dynamics:** The cockpit is connected to the base platform via a hexapod, which delivers high-frequency motions up to 30 Hz for all six degrees of freedom (DoF).

• **Performance and Modeling:** The simulator boasts a latency of less than 20 ms, which enables accurate stability assessments. Vehicle motion is generated using a real-time mathematical vehicle model, which produces the numerical output necessary to create the artificial environment.

• **Instrumentation and HMI:** The simulator is equipped with active Human-Machine (HM) interfaces, including the seat, safe belts, steering wheel, and brakes. It incorporates ECG and cameras for monitoring the driver. Additionally, the system supports the acquisition of data from supplementary sensors, such as the Instrumented Steering Wheel (ISW), EEG, and accelerometers, crucial for in-depth Human-Machine Interface (HMI) analysis.

• **Operational Use:** The system's wide working space makes it particularly suitable for reproducing demanding maneuvers, such as lane changes. Data regarding many vehicles, roads, and tracks are already available at the facility for testing relevant maneuvers, like sinusoidal steering with dwell time.

Theoretical and Experimental Findings on System Stability

The application of Bifurcation Theory to this non-linear vehicle-and-driver model provides a scientific taxonomy of unstable motions, highlighting why vehicle-centric, linear stability criteria (like those in ISO 8855) are insufficient.

INSTABILITY GOVERNED BY HOPF BIFURCATION

The project findings demonstrate that the driver's active control, particularly the inherent delay τ, fundamentally alters the stability characteristics of the system, even for vehicles considered inherently stable (like understeering ones).

• **Universal Critical Speed:** A key discovery is that a sufficiently high forward velocity always exists at which the controlled equilibrium of the vehicle-and-driver system becomes unstable through a **Hopf bifurcation**. This critical speed marks the threshold for stability loss and is present whether the vehicle is inherently oversteering (OV) or understeering (UN). For instance, a bare understeering vehicle (without a driver) is stable at any forward velocity, but the introduction of a driver creates a limit speed at which instability occurs [2].

• **The Role of Delay:** The driver's steering actuation delay τ is identified as the primary cause for the generation of these oscillatory unstable behaviors (limit cycles). Research shows that if the steering delay vanishes, the Hopf bifurcation is suppressed, meaning the characteristic loss of stability in the controlled system is oscillatory [5].

SADDLE-TYPE LIMIT CYCLES AND GLOBAL STABILITY

The Hopf bifurcation generates **limit cycles**, which represent periodic or oscillatory motions. For safety analysis, the unstable nature of these cycles is critical [6].

• **Saddle-Type Behavior:** The unstable cycles characterizing the motion of the vehicle-and-driver are consistently identified as **saddle-type limit cycles**. This saddle nature means the cycles possess a stable manifold (a surface of convergence) and an unstable manifold (a direction of repulsion).

• **Defining the Safety Threshold:** This stable manifold is critical because it defines the boundary of the **domain of attraction** (or basin of attraction) for the stable equilibrium (the intended trajectory). Any disturbance, if strong enough to push the system state outside this domain, leads to an uncontrolled motion (spin). Thus, the saddle limit cycle quantifies the "threshold above which a disturbance makes the motion unstable".

EXPERIMENTAL VALIDATION AND PRACTICAL METRICS

The theoretical predictions derived from the simplified model have been validated at the DriSMi facility, providing a good qualitative match.

• **Detection of Limit Cycles:** Experimental tests confirmed the existence of Hopf bifurcations and unstable limit cycles in the complex vehicle-and-driver system, demonstrating that instability arises when the trajectory approaches and is repelled by the saddle cycle [2,3].

• **Necessary and Sufficient Condition for Stability:** The concept of stability is simplified for practical application by relating it to the total energy of the vehicle. A **necessary and sufficient condition** for asymptotic stability is derived: a disturbance is controlled *if and only if* the variation of kinetic energy of the vehicle eventually vanishes. This finding is crucial as it demonstrates that vehicle stability can be effectively monitored by focusing on just two main state variables: **lateral speed v and yaw rate r**. This principle has been validated experimentally at the driving simulator [8,9].

• **Early Detection Strategy:** Building upon the properties of the saddle cycle, advanced strategies like the **Degree of Stability (DoS) criterion** have been developed to promptly detect unstable motion by projecting the system's current deviation onto the manifolds of the saddle cycle [6]. This criterion provides a quantitative stability index, indicating how close the system is to the stability threshold, that can be validated experimentally at the driving simulator.

Normative Impact and Proposed Revisions

The analysis conducted under the STAVE project highlights that the conceptual framework of stability within current international standards, specifically ISO 8855, **presents fundamental scientific limitations** that must be addressed to ensure the safe development and homologation of both conventional and automated road vehicles. The goal of STAVE is to provide the necessary scientific foundation to enable (National or) International Regulatory bodies to issue regulations or standards based on sound theoretical principles.

Proposed Revisions to ISO 8855 (Vocabulary and Conceptual Framework)

The existing ISO 8855:2011 standard defines stability primarily through local analysis around an equilibrium (steady state). The following revisions are necessary to align regulatory language with modern Vehicle System Dynamics and Systems Theory :

SHIFT FROM VEHICLE-CENTRIC TO SYSTEM-CENTRIC DEFINITIONS (ADDRESSING PROBLEM 1)

Stability must be fundamentally redefined as a property of the **Vehicle-and-Driver System** (human or non-human controller). The driver is a crucial component whose control action, particularly the inherent delay, can stabilize an unstable vehicle or, conversely, destabilize an inherently stable vehicle.

GENERALIZATION OF STABILITY TO TRAJECTORY/MOTION (ADDRESSING PROBLEM 2)

Stability definitions should be broadened to refer to the recovery of a **motion** or **trajectory**, rather than being strictly restricted to the steady state as defined in the regulation. A vehicle-and-driver system could be considerd stable if the sum of the applied external forces and moments and the inertial forces and moments which balance them form an unchanging force and moment system over an arbitrarily long time period.

• The definition of *Steady State* (ISO 8855: 12.2.1) should be generalized including the fact that also the driver should not introduce energy in the vehicle subsystem.

• The definition of *disturbance input* (ISO 8855: 11.1.2) should be generalized to include disturbances originating from necessary evasive manoeuvres performed by the driver.

• New definitions of *standard driving manoeuvres* should be introduced, explicitly including disturbances resulting from necessary evasive manoeuvres performed by the driver, so that the stability of such manoeuvres can be properly evaluated.

DESCRIPTION OF THE MOTION IN TERMS OF THE VEHICLE+DRIVER SYSTEMS (ADDRESSING PROBLEM 3)

ISO 8855 currently conflates stability characteristics (that in practice is the ability of the vehicle-and-driver system to do a specific manoeuvre) with the nature of the transient motion (oscillatory/non-oscillatory). **The two aspects should be treated separately**: in particular, the relationship between oscillatory behaviour and driver should be highlighted, to better allow the understanding of the phenomenon.

NECESSITY OF GLOBAL STABILITY AND ROBUSTNESS (ADDRESSING PROBLEM 4)

Stability must be quantified not just locally (following a *small* disturbance), but globally, by defining the **threshold** above which a disturbance causes the motion to become uncontrollable. This threshold directly relates to the safety index of the vehicle at a given speed.

• **Bifurcation Theory Application:** The appropriate methodology for quantifying this threshold is Bifurcation Theory, which uses full nonlinear models to assess large inputs (panic maneuvers, wind gusts).

• **The Domain of Attraction:** The critical stability measure is the **Domain of Attraction** (or basin of attraction). This domain is mathematically defined by the stable manifold of the unstable **saddle-type limit cycle** that surrounds the stable equilibrium. The unstable limit cycle is the defining feature of instability in the vehicle-and-driver system.

Stability Criteria for Regulation

The research provides a framework for defining scientifically sound regulatory criteria based on quantifiable dynamic phenomena:

• **Critical Speed Identification:** The existence of a critical speed (related to the Hopf bifurcation) at which the controlled equilibrium becomes unstable must be quantified for all vehicle types (understeering and oversteering), showing that instability is always possible due to driver delay.

• **Kinetic Energy Criterion:** A necessary and sufficient condition for asymptotic stability is derived: a disturbance is controlled if and only if the variation of the vehicle's kinetic energy (related to lateral speed and yaw rate) eventually vanishes. This allows stability assessment to focus on just two main state variables.

• **Prompt Detection Index (DoS):** Advanced tools like the *Degree of Stability (DoS) criterion* are proposed to promptly detect unstable motion by measuring the system's current proximity and trajectory relative to the unstable limit cycle, offering a quantitative stability index suitable for integration into future control algorithms and homologation test procedures.

Training and Education Mandate

Given the conceptual shortcomings in current terminology, STAVE considers it essential to provide a brief course for governmental staff on nonlinear vehicle dynamics and Bifurcation Theory, ensuring that homologation activities for automated vehicles (UN Regulation 157) are grounded in proper scientific understanding.

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